Circuit elements are categorized into:

Passive elements are:

1-Resistor المقاوم

2- Inductor اداة الحث

3- Capacitor مكثف

Active elements such as

1-Voltage source like batteries

2-Current source

3-Generators

4-Operational amplifier

There are two types of elements found in electric circuits: ***passive*** elements and ***active*** elements. An active element is capable of generating energy while a passive element is not.

1-Passive elements

1.1 Resistance of the material:

The flow of charge through any material encounters an opposing force due to the collisions between electrons and between electrons and other atoms in the material, ***which converts electrical energy into another form of energy such as heat***, is called the **resistance** of the material . The unit of measurement of resistance is the **ohm,** for which the symbol is (Ω)
The circuit symbol for resistance appears in Fig. (1.1)

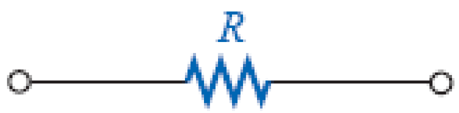


Fig. 1-1

Resistance symbol

The resistance of any material with a uniform cross-sectional area is determined by the following four factors:

1. Material resistivity
2. Length
3. Cross-sectional area
4. Temperature

Conductors will have low resistance levels, while insulators will have high resistance characteristics.

At a fixed temperature of 20°C (room temperature), the resistance is related to the other three factors by:

$$\boxed{R = \rho \frac{l}{A}} \qquad \text{(ohms, } \Omega\text{)} \qquad \text{\_\_\_\_\_\_\_\_\_\_(1.1)}$$

ρ:	ohm-centimeters
l:	centimeters
A:	square centimeters

Where ρ (Greek letter rho) is a characteristic of the material called the resistivity, l is the length of the sample, and A is the cross-sectional area of the sample.

1-2 (a) Temperature effect (T)

For good **conductors,** an increase in temperature will result in an increase in the resistance level. Consequently, conductors have a positive temperature coefficient. Fig (2-1-a)

For **semiconductor** materials, an increase in temperature will result in a decrease in the resistance level. Consequently, semiconductors have negative temperature coefficients. Fig (2-1-b)

As with semiconductors, an increase in temperature will result in a decrease in the resistance of an **insulator.** The result is a negative temperature coefficient.

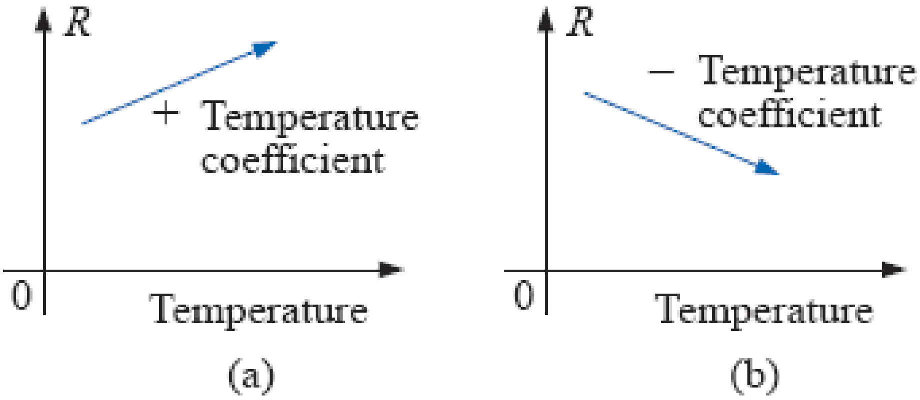


Fig 1-2
(a) Positive temperature coefficient for conductors.
(b) Negative temperature coefficient for semiconductors and insulators.

1-2(b) Inferred Absolute Temperature(درجة الحرارة المطلقة المُسْتَنْتَجة الصحيحة)

The proper inferred absolute temperature may be written as follows:

$$\boxed{\frac{|T_i| + T_1}{R_1} = \frac{|T_i| + T_2}{R_2}} \qquad \text{--------------------1.2)}$$

where |*Ti*| indicates that the inferred absolute temperature of the material involved is inserted as a positive value in the equation. In general, therefore, associate the sign only with $T1$ and $T2$.

EXAMPLE 1-1 If the resistance of a copper wire is 50 Ω at 20°C, what is its resistance at 100°C (boiling point of water)?
Note: Inferred absolute temperatures (Ti).of the copper is 234.5°

Solution:

$$\frac{234.5°\text{C} + 20°\text{C}}{50\ \Omega} = \frac{234.5°\text{C} + 100°\text{C}}{R_2}$$

$$R_2 = \frac{(50\ \Omega)(334.5°\text{C})}{254.5°\text{C}} = \mathbf{65.72\ \Omega}$$

EXAMPLE 1-2 If the resistance of aluminum wire at room temperature (20°C) is 100 mΩ(measured by a milliohm meter), at what temperature will its resistance increase to 120 mΩ?

Solution:

$$\frac{236°\text{C} + 20°\text{C}}{100\ \text{m}\Omega} = \frac{236°\text{C} + T_2}{120\ \text{m}\Omega}$$

$$T_2 = 120\ \text{m}\Omega\left(\frac{256°\text{C}}{100\ \text{m}\Omega}\right) - 236°\text{C}$$

$$T_2 = \mathbf{71.2°C}$$

Ex. 1-3 if the resistance of a copper wire at freezing (0°C) is 30 Ω what is its resistance at -40°C?

Sol.

$$\frac{234.5°\text{C} + 0}{30\ \Omega} = \frac{234.5°\text{C} - 40°\text{C}}{R_2}$$

$$R_2 = \frac{(30\ \Omega)(194.5°\text{C})}{234.5°\text{C}} = \mathbf{24.88\ \Omega}$$

1-3 CONDUCTANCE (G)

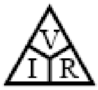
By finding the reciprocal of the resistance of a material, we have a measure of how well the material will conduct electricity. The quantity is called **conductance,** has the symbol *G*, and is measured in *Siemens* (S)**.**

$$\boxed{G = \frac{1}{R}} \qquad \text{(siemens, S)} \qquad \text{------------------ (1-3)}$$

In equation form, the conductance is determined by:

$$\boxed{G = \frac{A}{\rho l}} \qquad \text{(S)} \qquad \text{------------------ (1-4)}$$

1-4 Ohms law



$$\boxed{R = \frac{E}{I}} \qquad \text{(ohms, } \Omega\text{)}$$

Or

$$\boxed{I = \frac{E}{R}} \qquad \text{(amperes, A)}$$

Or

$$\boxed{E = IR} \qquad \text{(volts, V)}$$

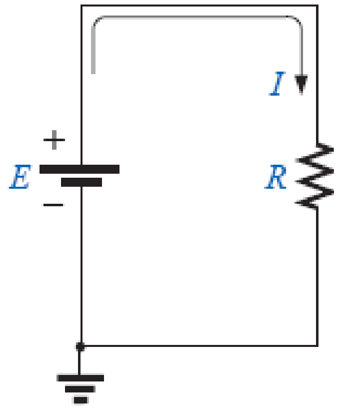


Fig 1-3

Basic circuit

Ex.1-4 Determine the current resulting from the application of a 9V battery across a network with a resistance of 2.2 Ω.

Sol.

$$I = \frac{E}{R} = \frac{9\text{ V}}{2.2\ \Omega} = \mathbf{4.09\ A}$$

Ex. 1-5 Calculate the voltage that must be applied across the soldering iron of Fig. 1-4 to establish a current of 1.5 A through the iron if its internal resistance is 80 Ω.

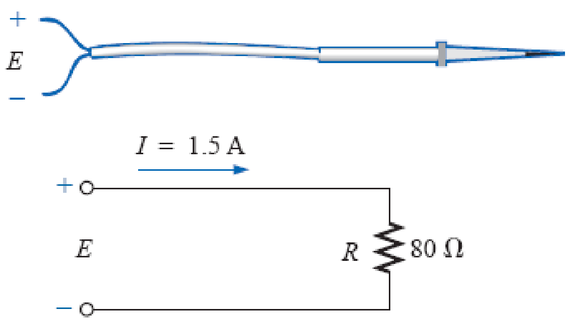


Fig1-4

Sol.

$$E = IR = (1.5\text{ A})(80\ \Omega) = \mathbf{120\ V}$$

1-5 POWER

power and energy calculations are important in circuit analysis.
Power is an indication of how much work (the conversion of energy from one form to another) can be done in a specified amount of time, that is, a *rate* of doing work. For instance, a large motor has more Power than a small motor because it can convert more electrical energy into mechanical energy in the same period of time. Since converted energy is measured in *joules* (J) and time in seconds (s), power is measured in joules/second (J/s). The electrical unit of measurement for power is the watt (W),

Power is the time rate of expending or absorbing energy, measured in watts (W).

$$\boxed{P = \frac{W}{t}} \qquad \text{(watts, W, or joules/second, J/s)}$$

$$\boxed{1 \text{ watt (W)} = 1 \text{ joule/second (J/s)}}$$

$$\boxed{1 \text{ horsepower} \cong 746 \text{ watts}}$$

$$P = \frac{W}{t} = \frac{QV}{t} = V\frac{Q}{t}$$

But $$I = \frac{Q}{t}$$

so that $$\boxed{P = VI} \qquad \text{(watts)}$$

$$P = VI = V\left(\frac{V}{R}\right)$$

and $$\boxed{P = \frac{V^2}{R}} \quad \text{(watts)}$$

or $$P = VI = (IR)I$$

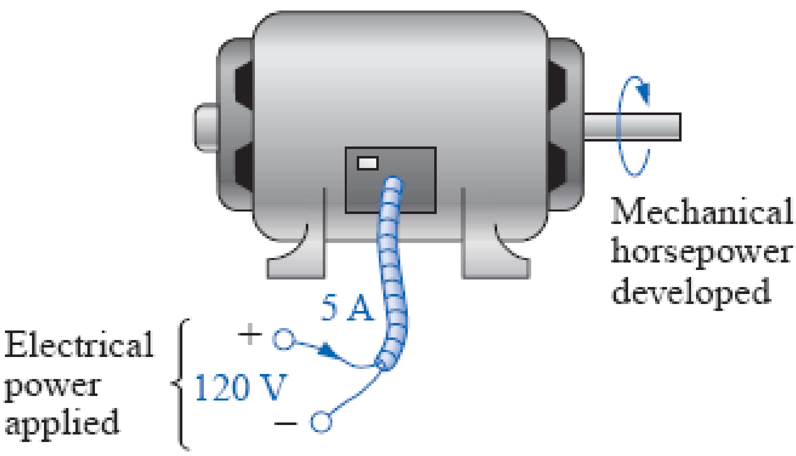
and $$\boxed{P = I^2R} \quad \text{(watts)}$$

The magnitude of the power delivered or absorbed by a battery is given by

$$\boxed{P = EI} \quad \text{(watts)}$$

With E the battery terminal voltage and I the current through the source

EXAMPLE 1-6 Find the power delivered to the dc motor of Fig 1-5:



Sol.

$$P = VI = (120\ \text{V})(5\ \text{A}) = 600\ \text{W} = \mathbf{0.6\ kW}$$

<u>EXAMPLE 1-7</u> what is the power dissipated by a 5Ω resistor if the current is 4 A?

<u>Sol.</u>

$$P = I^2R = (4\ \text{A})^2(5\ \Omega) = \mathbf{80\ W}$$

2- <u>Active elements</u>

2-1 <u>Voltage source and current source</u>

The most important active elements are voltage or current sources that generally deliver power to the circuit connected to them. There are two kinds of sources:

a. <u>Independent sourses</u>
b. <u>dependent sources</u>

> An ideal independent source is an active element that provides a specified voltage or current that is completely independent of other circuit variables.

An independent voltage source delivers to the circuit whatever current is necessary to maintain its terminal voltage batteries and generators may be regarded as approximations to ideal voltage sources. Figure 2-1 shows the symbols for independent voltage sources.

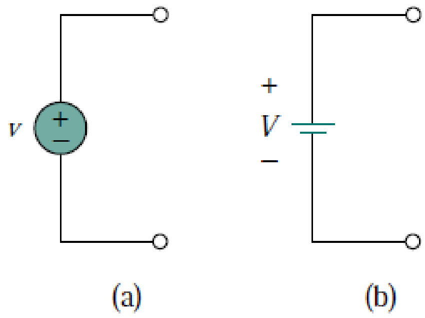


Fig2-1
Symbols for independent voltage sources:
(a) Used for constant or time-varying voltage,
(b) Used for constant voltage (dc).

An ideal independent current source is an active element that provides a specified current completely independent of the voltage across the source That is, the current source delivers to the circuit whatever voltage is necessary to maintain the designated current. The symbol for an independent current source is displayed in Fig. 2-2, where the arrow indicates the direction of current *i*.

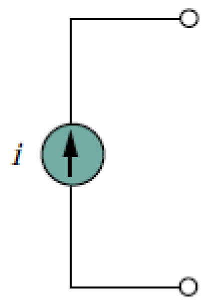


Fig2-2
Symbol for independent current source.

Dependent sources are usually designated by diamond-shaped symbols, as shown in Fig. 2-3.

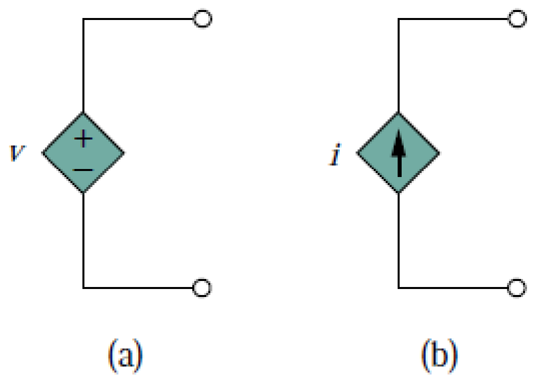


Fig 2-3
Symbols for:
(a) Dependent voltage source,
(b) Dependent current source

Dependent sources are useful in modeling elements such as transistors, operational amplifiers and integrated circuits.

An ideal dependent (or controlled) source is an active element in which the source quantity is controlled by another voltage or current.

NOTE

1- The term (ideal source) means that the internal resistance (R_s) of the source (voltage source or current source) equal zero.

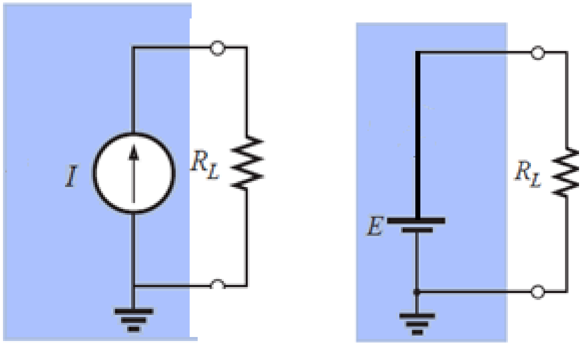


Fig 2-4

Ideal sources

2- The term (actual source) means that there is an internal resistance (R_s) of the source (voltage source or current source).

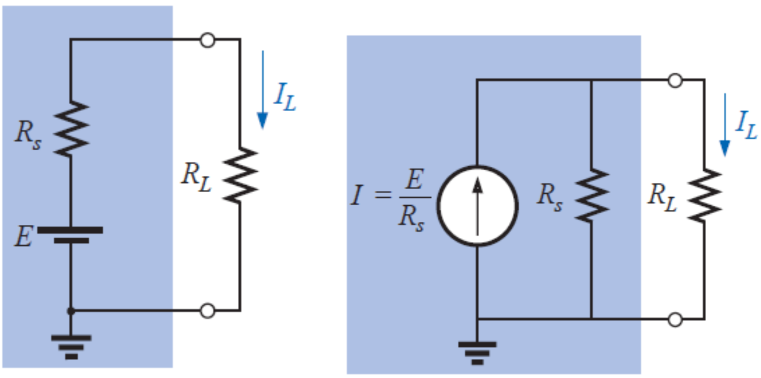


Fig 2-5

Actual sources

Please read and try understand in the first reference Chapter 1, 2, 3, and 4.

References:

1- **Introductory Circuits Analysis, By Boylested, Tenth (10th) Edition.**

2- **Schaum's Outline of Theory and Problems of Basic Circuit Analysis, By John O'Malley, Second (2nd) Edition.**

3- **Any reference that has Direct Current Circuits Analysis (DCCA).**

2- Actual sources

Voltage and Current sources

For the voltage source, if ***Rs* = 0** Ω or is so small compared to any series resistor that it can be ignored, then we have an **"ideal" voltage** source. For the current source, if ***Rs*** = ∞ or is large enough compared to other parallel elements that it can be ignored, and then we have an **"ideal" current** source. See fig (2-4).
If the internal resistance is included with either source, then we have an **"actual"** voltage source or **"actual"** current source fig (2-5); then that source can be converted to the other type. Fig (2-6).

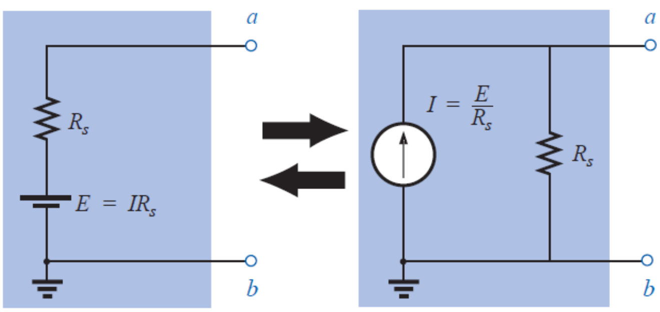


Fig (2-6)

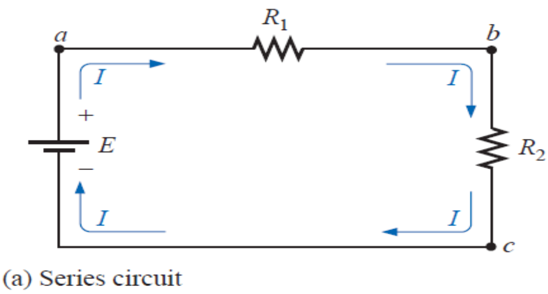
Source conversion.

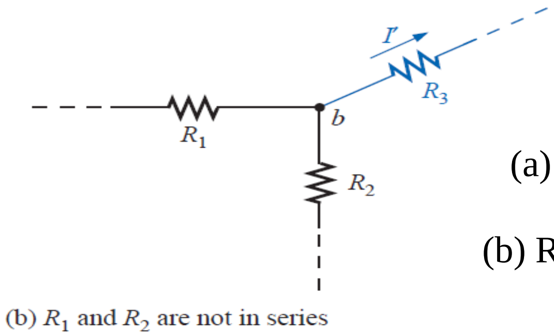
Voltage source to current source and vice versa

3- Network simplification

3-1 SERIES CIRCUITS

A circuit consists of any number of elements joined at terminal points, providing at least one closed path through which charge can flow. The circuit of Fig. 3-1 has three elements joined at three terminal points (*a*, *b*, and *c*) to provide a closed path for the current *I*.





Fig(3-1)

(a) Series circuit R1and R2 and E

(b) R_1 and R_2 and R_3 are **not** in series.

In Fig.(3-1) the resistors *R*1 and *R*2 are in series , the battery *E* and resistor *R*1 are in series, and the resistor *R*2 and the battery *E* are in series .Since all the elements are in series, the network is called a (**series circuit.)**

☺ **Note**

1- The total resistance of a series circuit is the sum of the resistance levels.

$RT= R_1+R_2$ (ohm Ω)

2- The current is the same through each element and the current drawn from the source (Total current I_T) of Fig. (3-1a) equal:

$I=I_{R1}=I_{R2}=I_T$ (Amp)

I_T can be determined using Ohm's law.

$I= I_T = \frac{E}{RT}$ (Amp)

3- $V_1=IR_1$ $V_2=IR_2$ $V_T= V_1+V_2$ (Volt)

4- The power delivered to each resistor can then be determined using any one of three equations:

$$\boxed{P_1 = V_1 I_1 = I_1^2 R_1 = \frac{V_1^2}{R_1}} \qquad \text{(watts, W)}$$

The power delivered by the source is

$$\boxed{P_{\text{del}} = EI} \qquad \text{(watts, W)}$$

The total power delivered to a resistive circuit is equal to the total power dissipated by the resistive elements.

$$\boxed{P_{\text{del}} = P_1 + P_2 + P_3 + \cdots + P_N}$$

EXAMPLE 1

a. Find the total resistance for the series circuit of Fig. 3-2

b. Calculate the source current *Is*.

c. Determine the voltages *V*1, *V*2, and *V*3.

d. Calculate the power dissipated by *R*1, *R*2, and *R*3.

e. Determine the power delivered by the source, and compare it to the sum of the power levels of part (d).

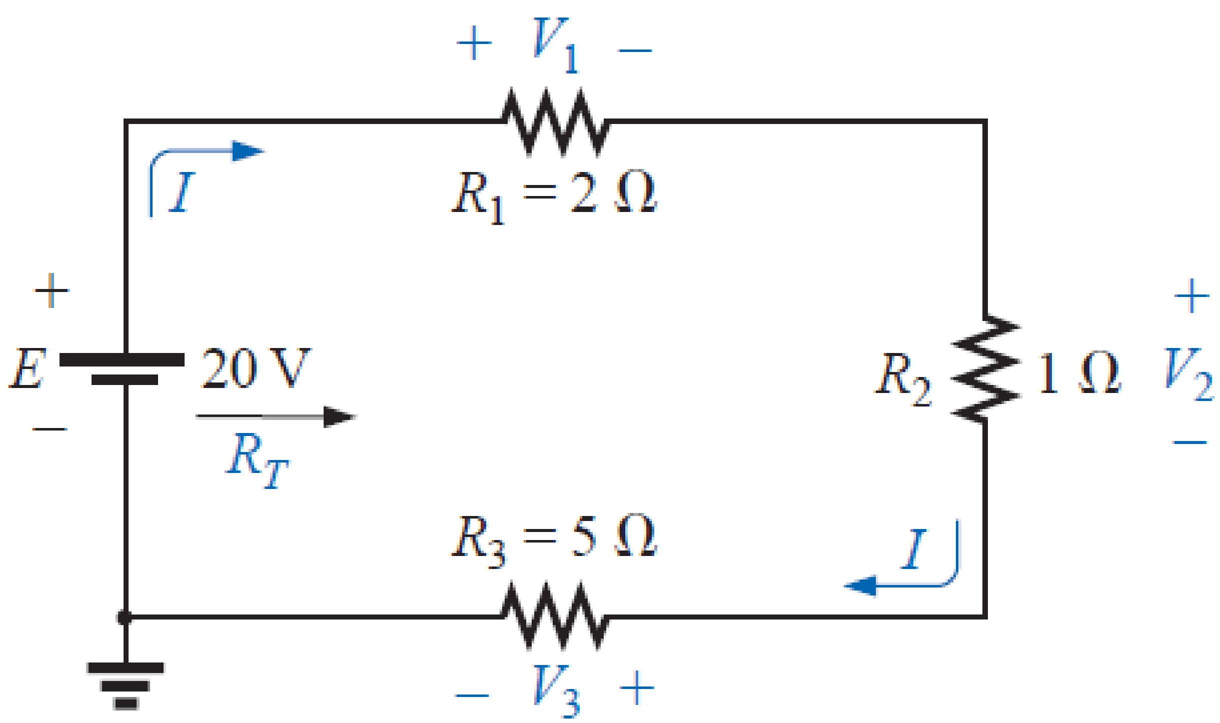


Fig (3-2)

Solutions:

a. $R_T = R_1 + R_2 + R_3 = 2\ \Omega + 1\ \Omega + 5\ \Omega = \mathbf{8\ \Omega}$

b. $I_s = \dfrac{E}{R_T} = \dfrac{20\text{ V}}{8\ \Omega} = \mathbf{2.5\ A}$

c. $V_1 = IR_1 = (2.5\text{ A})(2\ \Omega) = \mathbf{5\ V}$
$V_2 = IR_2 = (2.5\text{ A})(1\ \Omega) = \mathbf{2.5\ V}$
$V_3 = IR_3 = (2.5\text{ A})(5\ \Omega) = \mathbf{12.5\ V}$

d. $P_1 = V_1I_1 = (5\text{ V})(2.5\text{ A}) = \mathbf{12.5\ W}$
$P_2 = I_2^2R_2 = (2.5\text{ A})^2(1\ \Omega) = \mathbf{6.25\ W}$
$P_3 = V_3^2/R_3 = (12.5\text{ V})^2/5\ \Omega = \mathbf{31.25\ W}$

e. $P_{del} = EI = (20\text{ V})(2.5\text{ A}) = \mathbf{50\ W}$
$P_{del} = P_1 + P_2 + P_3$
$50\text{ W} = 12.5\text{ W} + 6.25\text{ W} + 31.25\text{ W}$
$\underline{50\text{ W} = 50\text{ W} \quad \text{(checks)}}$

EXAMPLE 2 Determine *RT*, *I*, and *V*2 for the circuit of Fig. 3-3

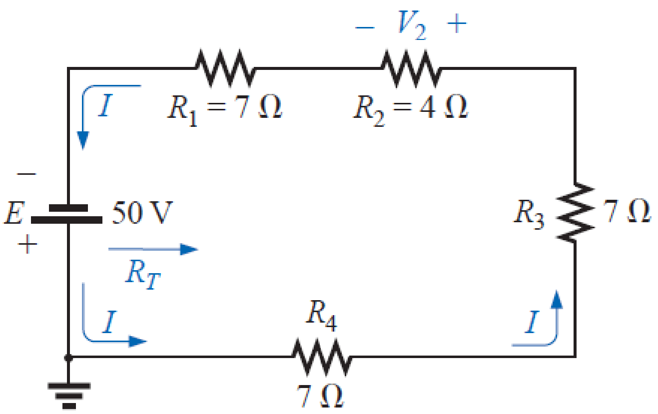


Fig 3-3

Solution. Note the current direction as established by the battery and the polarity of the voltage drops across R_2 as determined by the current direction

$R_T = R_1 + R_2 + R_3 + R_4$

$R_T = 7+4+7+7 = 25\ \Omega$

$$I = \frac{E}{R_T} = \frac{50\text{ V}}{25\ \Omega} = \mathbf{2\ A}$$

$$V_2 = IR_2 = (2\text{ A})(4\ \Omega) = \mathbf{8\ V}$$

EXAMPLE 3 Given R_T and I, calculate R_1 and E for the circuit of Fig.3-4 .

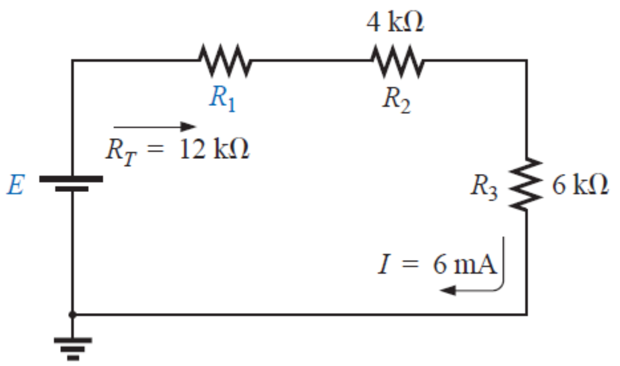
Solution:

$$R_T = R_1 + R_2 + R_3$$

$$12\text{ k}\Omega = R_1 + 4\text{ k}\Omega + 6\text{ k}\Omega$$

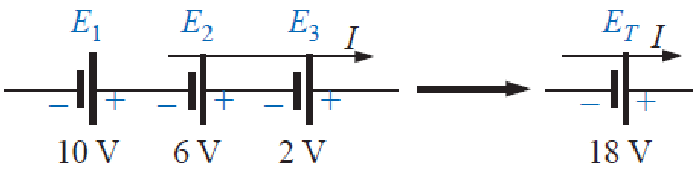
$$R_1 = 12\text{ k}\Omega - 10\text{ k}\Omega = \mathbf{2\ k\Omega}$$

$$E = IR_T = (6 \times 10^{-3}\text{ A})(12 \times 10^3\ \Omega) = \mathbf{72\ V}$$



Fig(3-4)

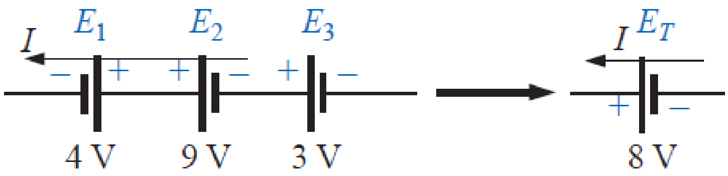
3-2 VOLTAGE SOURCES IN SERIES



(a)

$ET = E1 + E2 + E3 = 10\text{ V} + 6\text{ V} + 2\text{ V} = 18\text{ V}$

and the polarity shown in the figure



(b)

$E_T = E_2 + E_3 - E_1 = 9\text{ V} + 3\text{ V} - 4\text{ V} = 8\text{ V}$

and the polarity shown in the figure

Fig3.5
(a ,b) Reducing series dc voltage sources to a single source.

3-3 KIRCHHOFF'S VOLTAGE LAW

Kirchhoff's voltage law (KVL) states that the algebraic sum of the potential rises and drops around a closed loop (or path) is zero.

A closed loop is any continuous path that leaves a point in one direction and returns to that same point from another direction without leaving the circuit.

The clockwise (CW) direction will be used throughout the text for all applications of Kirchhoff's voltage law. Be aware, however, that the same result will be obtained if the counterclockwise (CCW) direction is chosen and the law applied correctly. A plus sign is assigned to a potential rise (- to +), and a minus sign to a potential drop (+ to -). If we follow the current in Fig. (3-6)rom point a, we first encounter a potential drop V1 (+ to -) across R1 and then another potential drop V2 across R2.

Continuing through the voltage source, we have a potential rise E (- to +) before returning to point a. In symbolic form, where Σ represents (summation), the closed loop, and V the potential **drops** and **rises**, we have :

$$\boxed{\Sigma_{\circlearrowright} V = 0}$$ (Kirchhoff's voltage law in symbolic form)

Which for the circuit of Fig. (3-6) yields (clockwise direction, following the current I and starting at point d):

$$+E - V_1 - V_2 = 0$$

$$E = V_1 + V_2$$

the applied voltage of a series circuit equals the sum of the voltage drops across the series elements.

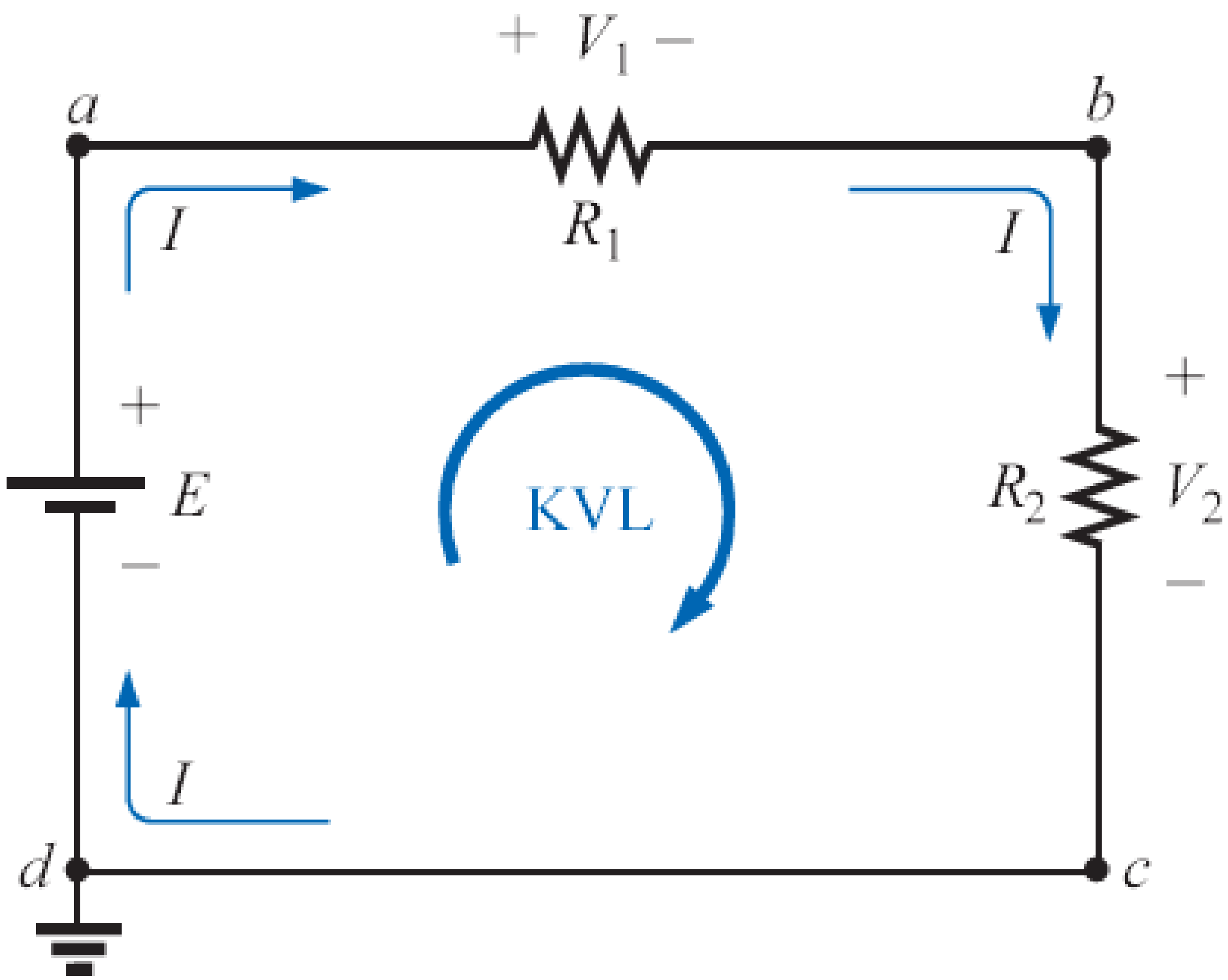


Fig (3-6) Applying Kirchhoff's voltage law to a series dc circuit.

Kirchhoff's voltage law can also be stated in the following form:

$$\boxed{\sum\nolimits_{\circlearrowright} V_{\text{rises}} = \sum\nolimits_{\circlearrowright} V_{\text{drops}}}$$

EXAMPLE 4 Determine the unknown voltages for the networks of Fig. (3-7)

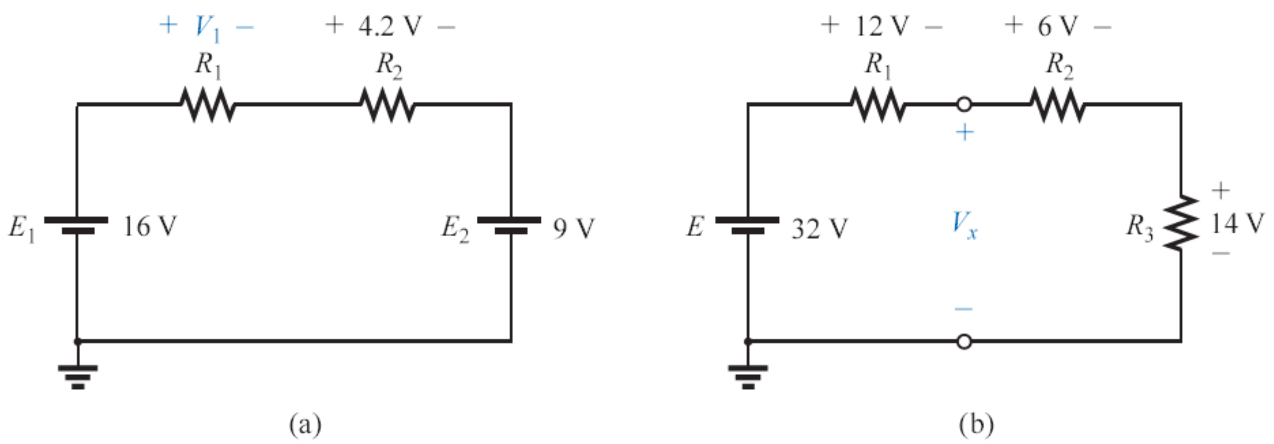


Fig3-7

Sol:

a-

$$+E_1 - V_1 - V_2 - E_2 = 0$$

and $$V_1 = E_1 - V_2 - E_2 = 16\text{ V} - 4.2\text{ V} - 9\text{ V}$$
$$= \mathbf{2.8\ V}$$

b-

$$+E - V_1 - V_x = 0$$

and
$$V_x = E - V_1 = 32\text{ V} - 12\text{ V}$$
$$= \mathbf{20\ V}$$

Using the clockwise direction for the other loop involving R_2 and R_3 will result in

$$+V_x - V_2 - V_3 = 0$$

and
$$V_x = V_2 + V_3 = 6\text{ V} + 14\text{ V}$$
$$= \mathbf{20\ V}$$

EXAMPLE 5 For the circuit of Fig. 3-8

a. Find R_T.
b. Find I.
c. Find V_1 and V_2.
d. Find the power to the 4-Ω and 6-Ω resistors.
e. Find the power delivered by the battery, and compare it to that dissipated by the 4-Ω and 6-Ω resistors combined.
f. Verify Kirchhoff's voltage law (clockwise direction).

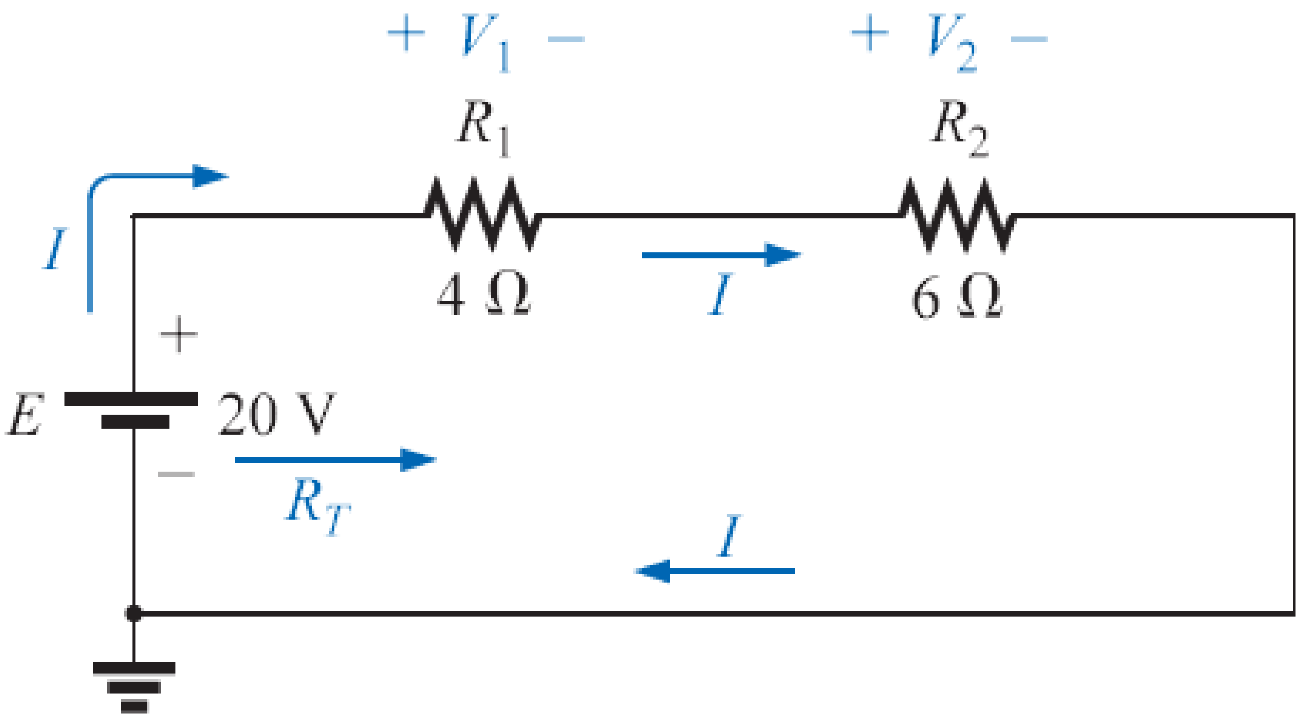


Fig 3-8

Solutions:

a. $R_T = R_1 + R_2 = 4\ \Omega + 6\ \Omega = \mathbf{10\ \Omega}$

b. $I = \dfrac{E}{R_T} = \dfrac{20\text{ V}}{10\ \Omega} = \mathbf{2\ A}$

c. $V_1 = IR_1 = (2\text{ A})(4\ \Omega) = \mathbf{8\ V}$
$V_2 = IR_2 = (2\text{ A})(6\ \Omega) = \mathbf{12\ V}$

d. $P_{4\Omega} = \dfrac{V_1^2}{R_1} = \dfrac{(8\text{ V})^2}{4} = \dfrac{64}{4} = \mathbf{16\ W}$

$P_{6\Omega} = I^2R_2 = (2\text{ A})^2(6\ \Omega) = (4)(6) = \mathbf{24\ W}$

e. $P_E = EI = (20\text{ V})(2\text{ A}) = \mathbf{40\ W}$
$P_E = P_{4\Omega} + P_{6\Omega}$
$40\text{ W} = 16\text{ W} + 24\text{ W}$
$40\text{ W} = 40\text{ W}$ (checks)

f. $\Sigma_{\circlearrowright} V = +E - V_1 - V_2 = 0$
$E = V_1 + V_2$
$20\text{ V} = 8\text{ V} + 12\text{ V}$
$20\text{ V} = 20\text{ V}$ (checks)

EXAMPLE 6 For the circuit of Fig. 3-9

a. Determine V_2 using Kirchhoff's voltage law.
b. Determine I.
c. Find R_1 and R_3.

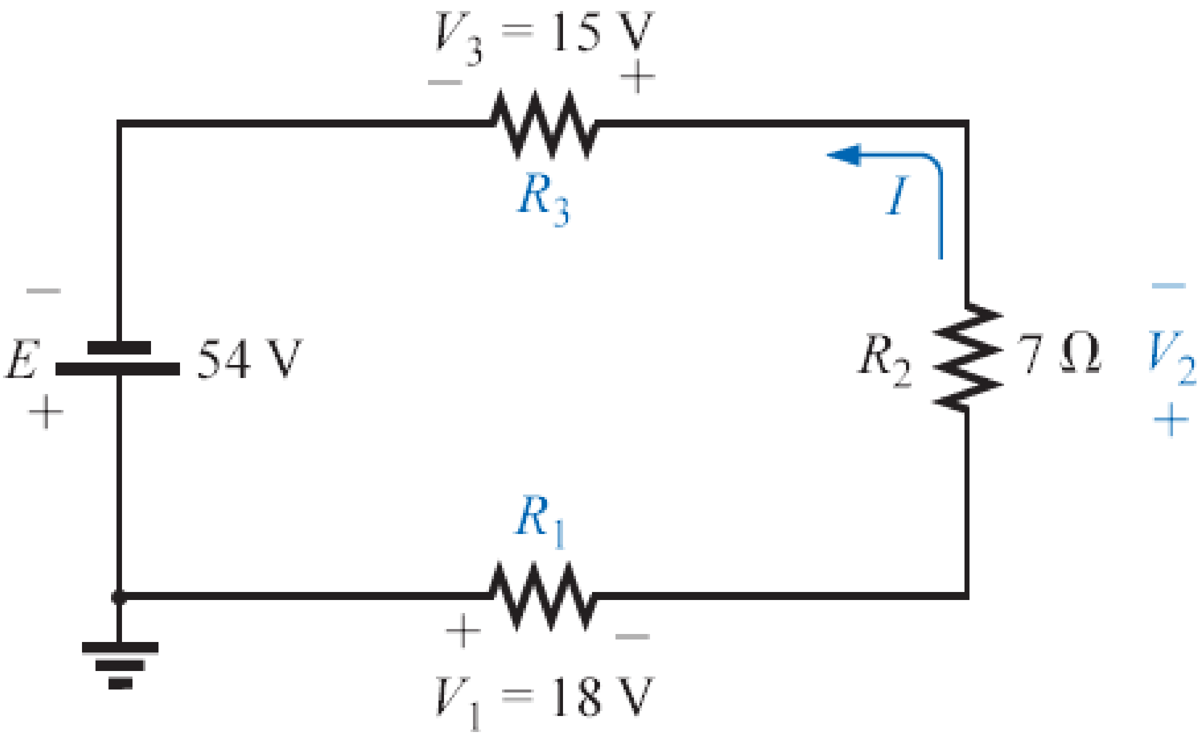


Fig 3-9

Solutions:

a. Kirchhoff's voltage law (clockwise direction):

$$-E + V_3 + V_2 + V_1 = 0$$

or $$E = V_1 + V_2 + V_3$$

and $$V_2 = E - V_1 - V_3 = 54\text{ V} - 18\text{ V} - 15\text{ V} = \mathbf{21\ V}$$

b. $I = \dfrac{V_2}{R_2} = \dfrac{21\text{ V}}{7\ \Omega} = \mathbf{3\ A}$

c. $R_1 = \dfrac{V_1}{I} = \dfrac{18\text{ V}}{3\text{ A}} = \mathbf{6\ \Omega}$

$R_3 = \dfrac{V_3}{I} = \dfrac{15\text{ V}}{3\text{ A}} = \mathbf{5\ \Omega}$

3-4 INTERCHANGING SERIES ELEMENTS

The elements of a series circuit can be interchanged without affecting the total resistance, current, or power to each element. For instance, the network of Fig. (3-10) can be redrawn as shown in Fig.(3-11)

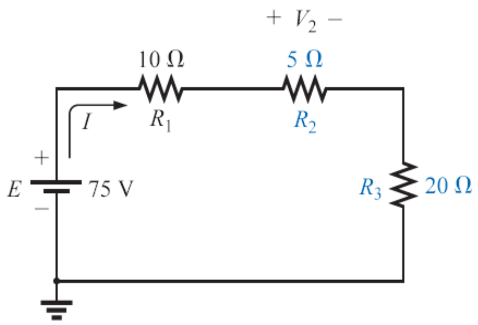


Fig. (3-10)

Series dc circuit with

elements to be interchanged

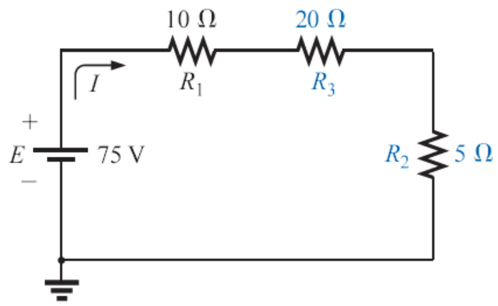


Fig. (3-11)

Circuit of

with R2and R3 interchanged.

EXAMPLE 7 Determine I and the voltage across the 7Ω resistor for the network of Fig. 3-12

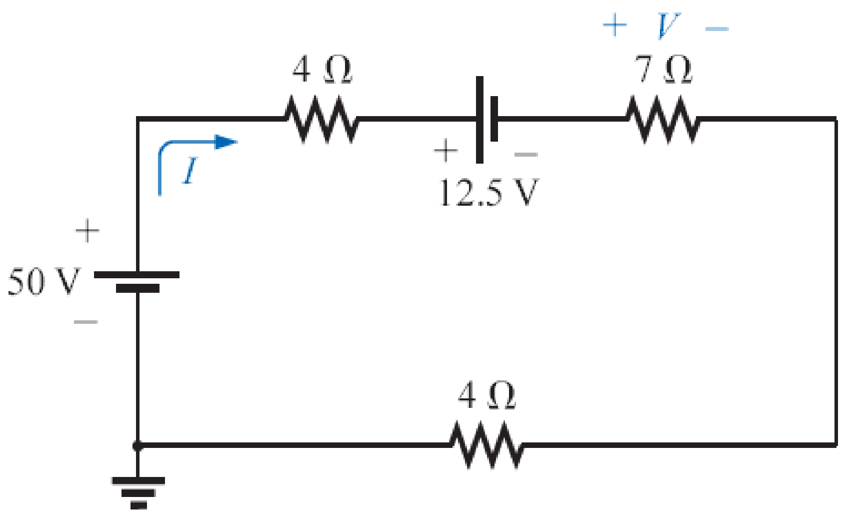


Fig 3-12

Solution:

The network is redrawn in Fig. 3-13

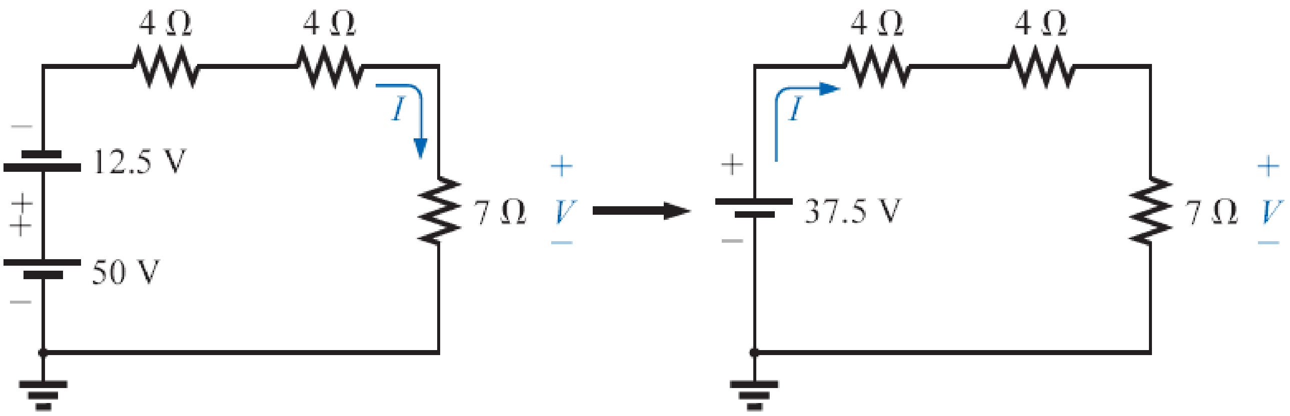


Fig 3-13

$$R_T = (2)(4\ \Omega) + 7\ \Omega = 15\ \Omega$$

$$I = \frac{E}{R_T} = \frac{37.5\ \text{V}}{15\ \Omega} = \mathbf{2.5\ A}$$

$$V_{7\Omega} = IR = (2.5\ \text{A})(7\ \Omega) = \mathbf{17.5\ V}$$

3-5 VOLTAGE DIVIDER RULE (V.D.R.)

The voltage across a resistor in a series circuit is equal to the value of that resistor times the total impressed voltage across the series elements divided by the total resistance of the series elements.

$$\boxed{V_x = \frac{R_x E}{R_T}} \qquad \text{(voltage divider rule)}$$

EXAMPLE 8 Determine the voltage V1 for the network of Fig. 3-14

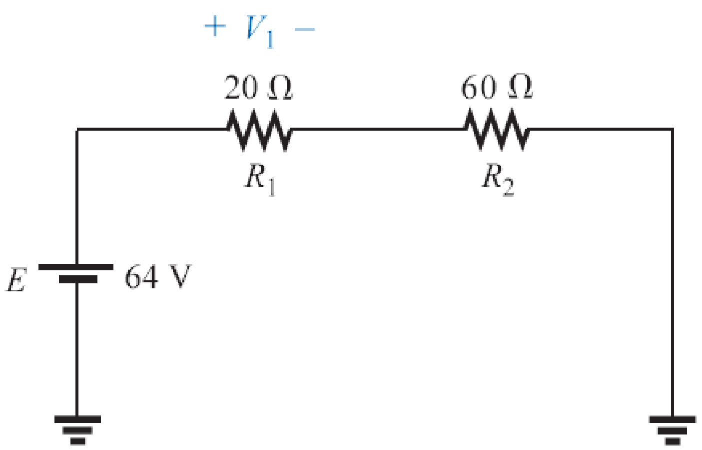


Fig (3-14)

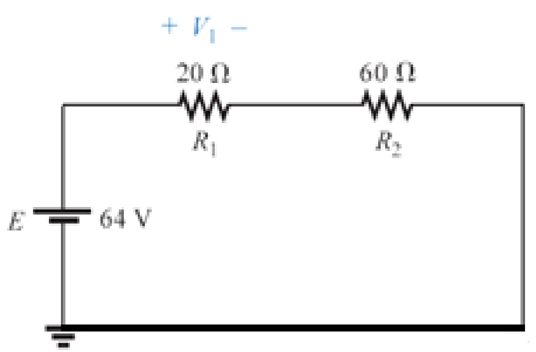


Fig (3-15)

Sol.

The circuit is simplified to fig (3-15)

$$V_1 = \frac{R_1 E}{R_T} = \frac{R_1 E}{R_1 + R_2} = \frac{(20\ \Omega)(64\ \text{V})}{20\ \Omega + 60\ \Omega} = \frac{1280\ \text{V}}{80} = \mathbf{16\ V}$$

<u>EXAMPLE 9</u> Using the voltage divider rule, determine the voltages V1 and V3 for the series circuit of Fig. 3-16

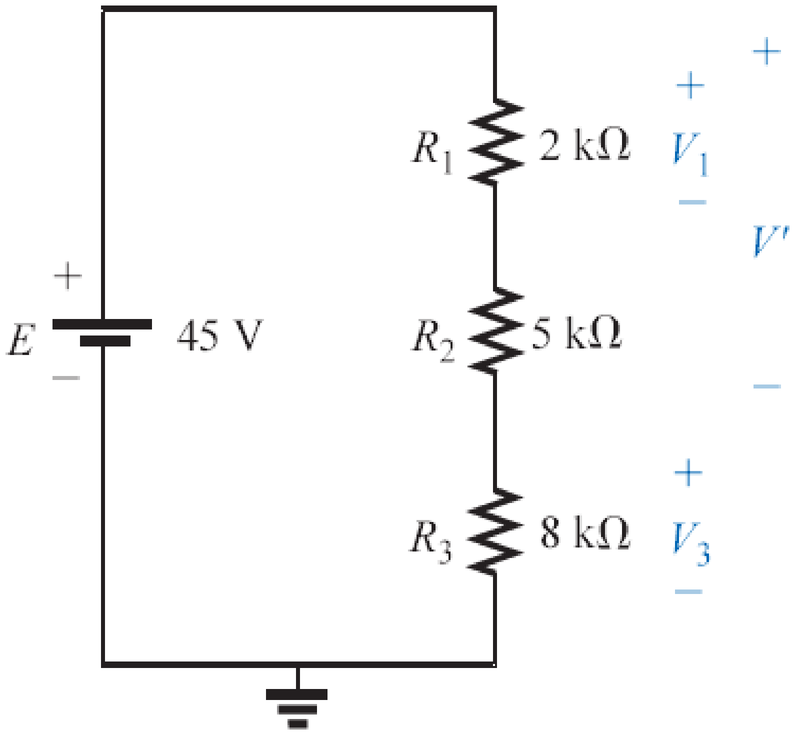


Fig 3-16

Solution:

$$V_1 = \frac{R_1E}{R_T} = \frac{(2\ \text{k}\Omega)(45\ \text{V})}{2\ \text{k}\Omega + 5\ \text{k}\Omega + 8\ \text{k}\Omega} = \frac{(2\ \text{k}\Omega)(45\ \text{V})}{15\ \text{k}\Omega}$$

$$= \frac{(2 \times 10^3\ \Omega)(45\ \text{V})}{15 \times 10^3\ \Omega} = \frac{90\ \text{V}}{15} = \mathbf{6\ V}$$

$$V_3 = \frac{R_3E}{R_T} = \frac{(8\ \text{k}\Omega)(45\ \text{V})}{15\ \text{k}\Omega} = \frac{(8 \times 10^3\ \Omega)(45\ \text{V})}{15 \times 10^3\ \Omega}$$

$$= \frac{360\ \text{V}}{15} = \mathbf{24\ V}$$

<u>Note</u> The rule can be extended to the voltage across two or more series elements.

$$\boxed{V' = \frac{R'E}{R_T}} \quad \text{(volts)}$$

<u>EXAMPLE 10</u> Determine the voltage V′ in Fig.(3-16) across resistors R_1 and R_2.

Solution:

$$V' = \frac{R'E}{R_T} = \frac{(2\ \text{k}\Omega + 5\ \text{k}\Omega)(45\ \text{V})}{15\ \text{k}\Omega} = \frac{(7\ \text{k}\Omega)(45\ \text{V})}{15\ \text{k}\Omega} = \mathbf{21\ V}$$

Please read and try understand in the first reference Chapter 5.

References:

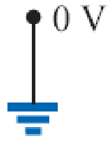
1- Introductory Circuits Analysis, By Boylested, Tenth (10th) Edition.

2- Schaum's Outline of Theory and Problems of Basic Circuit Analysis, By John O'Malley, Second (2nd) Edition.

3- Any reference that has Direct Current Circuits Analysis (DCCA).

Voltage Sources and Ground

The symbol for the ground connection appears in Fig. (1) With its defined



Fig(1)

Ground potential

If we take the circuit of fig (2)

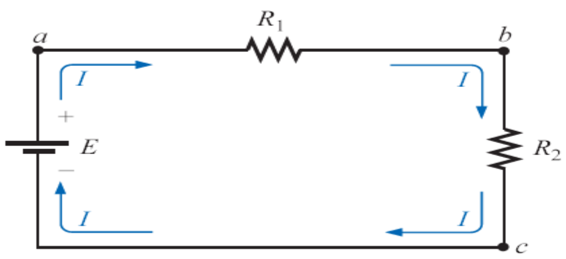


Fig (2)

If Fig. (2) Is redrawn with a grounded supply, it might appear as shown in Fig. 3(a), (b), or (c). In any case, it is understood that the negative terminal of the battery and the bottom of the resistor R_2 are at ground potential.

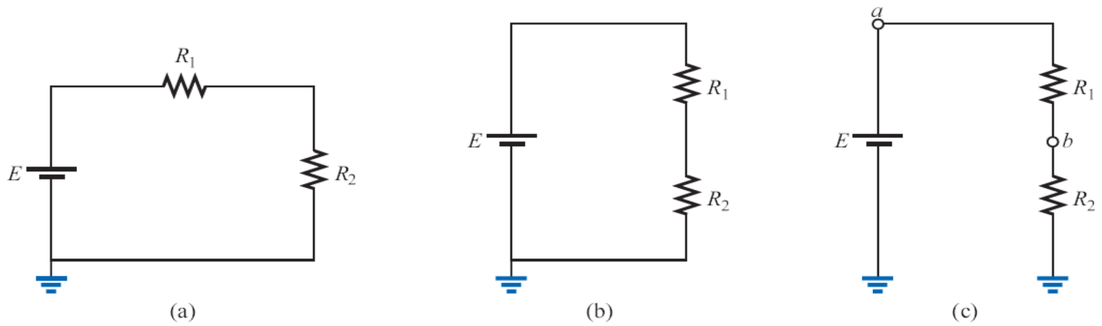


Fig (3) (a,b,c)

Three ways to sketch the same series dc circuit.

Example 1 Design a circuit by using the voltage divider rule of fig (4) such that ($V_{R1} = 4V_{R2}$).

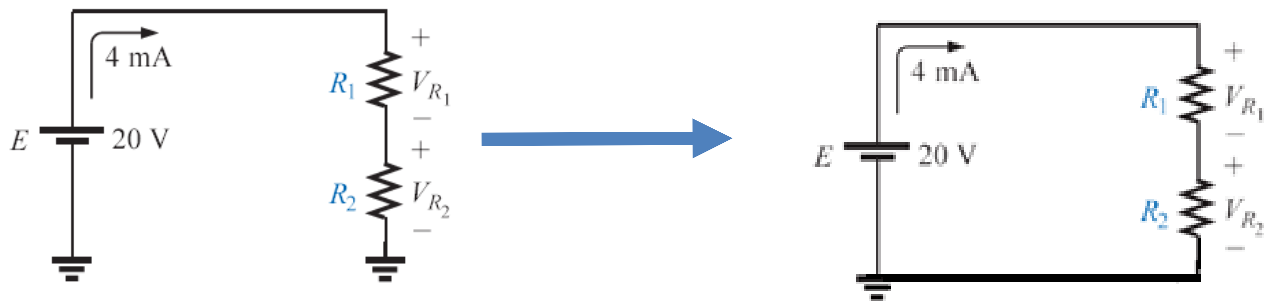


Fig (4)

Solution: The total resistance is defined by:

$$R_T = \frac{E}{I} = \frac{20\text{ V}}{4\text{ mA}} = 5\text{ k}\Omega$$

Since $V_{R_1} = 4V_{R_2}$,

$$R_1 = 4R_2$$

Thus $$R_T = R_1 + R_2 = 4R_2 + R_2 = 5R_2$$

and $$5R_2 = 5\text{ k}\Omega$$
$$R_2 = \mathbf{1\text{ k}\Omega}$$

and $$R_1 = 4R_2 = \mathbf{4\text{ k}\Omega}$$

Voltage sources may be indicated as shown in Figs. 5(a) and 6(a) rather than as illustrated in Fig. 5(b) and 6(b).

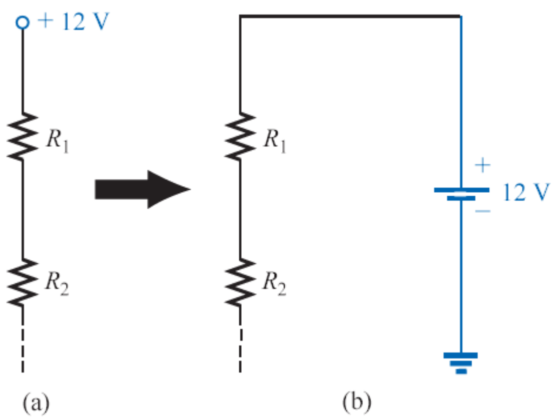


Fig (5)

Replacing the special notation for a positive dc voltage source with the standard symbol

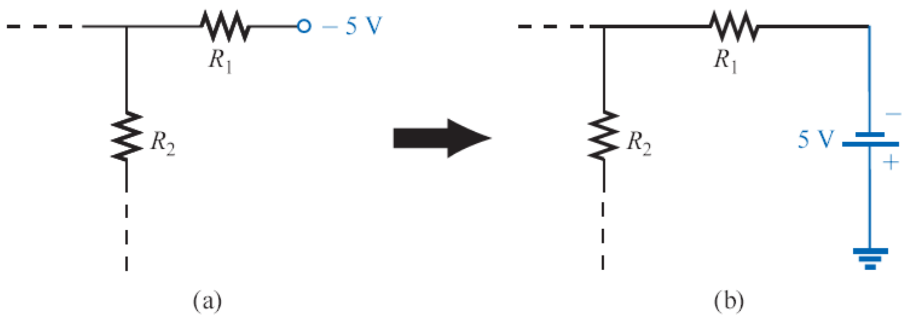


Fig (6)

Replacing the notation for a negative dc supply with the standard symbol.

Double-Subscript Notation

The double-subscript notation V_{ab} specifies point (a) as the higher potential. If this is not the case, negative sign must be associated with the magnitude of V_{ab}.

In other words,

The voltage V_{ab} is the voltage at point (a) ***with respect to (w.r.t.)*** *point (b).*

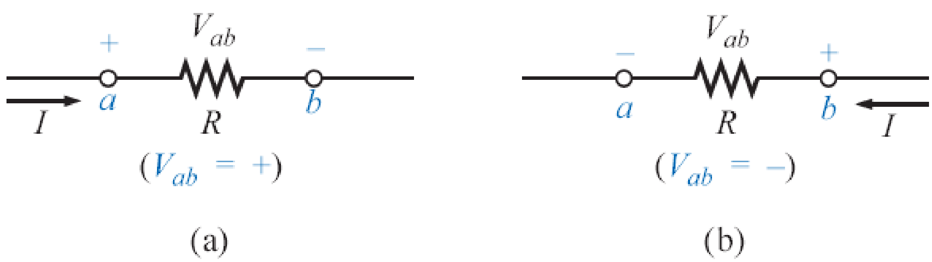


Fig (7)

The fact that voltage is an across variable and exists between two points has resulted in a double-subscript notation that defines the first subscript as the higher potential. In Fig. 7(a), the two points that define the voltage across the resistor R are denoted by (a and b). Since (a) is the first subscript for V_{ab}, point (a) must have a higher potential than point (b)if V_{ab} is to have a positive value. If, point (b) is at a higher potential than point (a) ,V_{ab} will have a negative value, as indicated in Fig. 7(b).

Single-Subscript Notation

The single-subscript notation (V_a) specifies the voltage at point a with respect to ground (Zero volts).

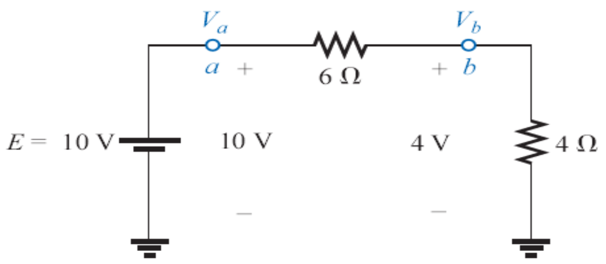


Fig. (8)

Defining the use of single-subscript notation

In Fig. 8, V_a is the voltage from point (a) to ground. In this case it is obviously 10 V since It is right across the source voltage E. The voltage V_b is the voltage from point (b) to ground. Because it is directly across the 4**Ω** resistor, V_b = 4 V.

The following relationship exists:

$$\boxed{V_{ab} = V_a - V_b} \qquad (1)$$

So from the equation (1):

$$V_{ab} = V_a - V_b = 10\text{ V} - 4\text{ V}$$
$$= 6\text{ V}$$

EXAMPLE 2 Find the voltage V_{ab} for the conditions of Fig. (9)

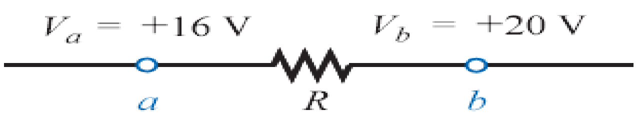


Fig (9)

Solution: Applying Eq. (1)

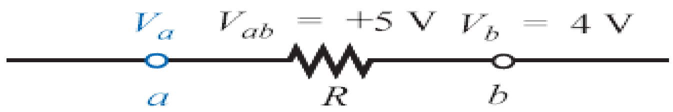
$V_{ab} = V_a - V_b = 16\text{ V} - 20\text{ V}$

$= -4\text{ V}$

☺ ***Note:*** ***the negative sign means that point (b) is at a higher potential than point (a).***

EXAMPLE 3 Find the voltage V_a for the configuration of Fig. (10)

Solution: Applying Eq. (1):



Fig(10)

$Vab = V_a - V_b$

and ***$V_a = V_{ab} + V_b = 5\text{ V} + 4\text{ V}$***

$= 9\text{ V}$

EXAMPLE 4 Find the voltage V_{ab} for the configuration of Fig. (11)

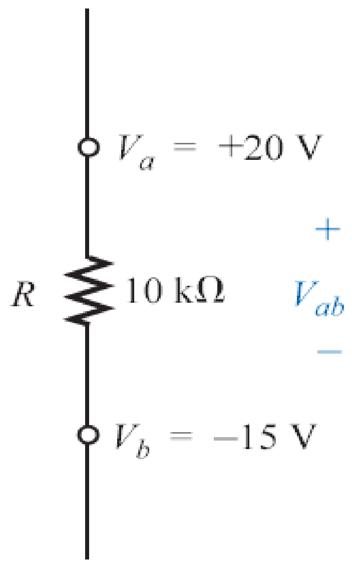


Fig (11)

Solution: Applying Eq. (1):

$$V_{ab} = V_a - V_b = 20\ V - (-15\ V) = 20\ V + 15\ V$$

$$= \quad 35\ V$$

<u>EXAMPLE 5</u> Find the voltages V_b, V_c, and V_{ac} for the network of Fig (12).

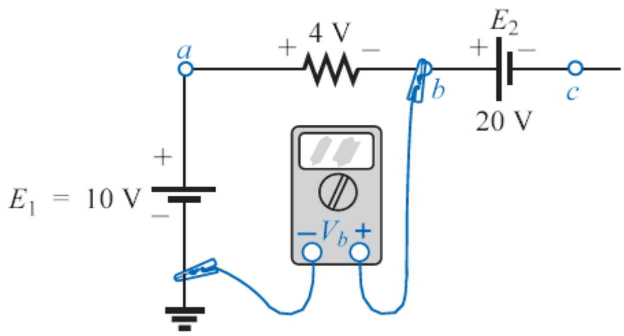


Fig (12)

<u>Solution</u>: Starting at ground potential (zero volts), we proceed through a rise of 10 V to reach point (a) and then pass through(a) drop in potential of 4 V to point (b). The result is that the meter will read:

$$V_b = +10\ V - 4\ V = 6\ V$$

$$V_c = Vb - 20\ V = 6\ V - 20\ V = -14\ V$$

$$V_{ac} = Va - Vc = 10\ V - (-14\ V)$$

$$= 24\ V$$

<u>EXAMPLE 6</u> Determine V_{ab}, V_{cb}, and V_c for the network of Fig. (13)

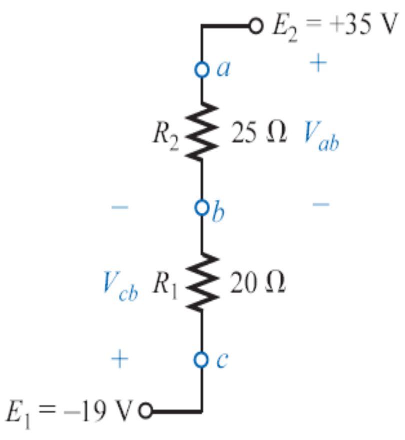


Fig (13)

Redraw the network as shown in Fig. (14)

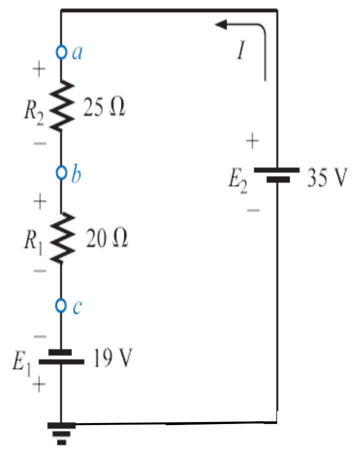


Fig (14)

<u>Sol</u>:

$$I = \frac{54\text{ V}}{45\ \Omega} = 1.2\text{ A}$$

$$V_{ab} = IR_2 = (1.2\text{ A})(25\ \Omega) = \mathbf{30\ V}$$

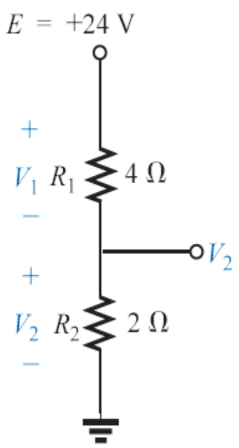
$$V_{cb} = -IR_1 = -(1.2\text{ A})(20\ \Omega) = \mathbf{-24\ V}$$

$$V_c = E_1 = \mathbf{-19\ V}$$

E_T= 19+35= 54 V

R_T = 20+25 =45

<u>EXAMPLE 7</u> Using the voltage divider rule, determine the voltagesV1 and V2 of Fig. (15).



Redraw the network as shown in Fig. (16)

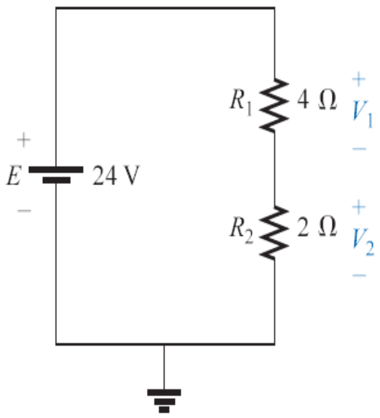


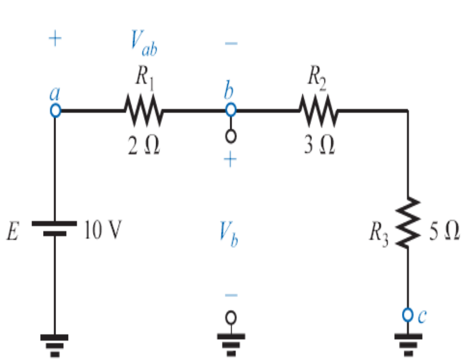
Fig (15)

fig (16)

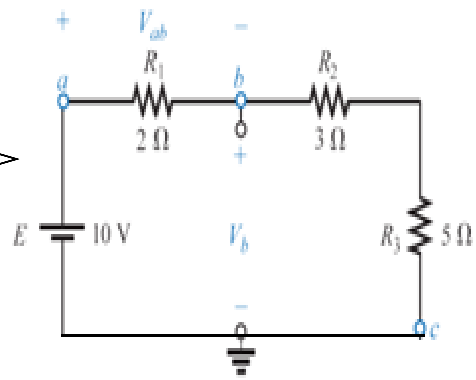
$$V_1 = \frac{R_1 E}{R_1 + R_2} = \frac{(4\ \Omega)(24\ \text{V})}{4\ \Omega + 2\ \Omega} = \mathbf{16\ V}$$

$$V_2 = \frac{R_2 E}{R_1 + R_2} = \frac{(2\ \Omega)(24\ \text{V})}{4\ \Omega + 2\ \Omega} = \mathbf{8\ V}$$

EXAMPLE 8 For the network of Fig. (17)



Redraw the network as shown in Fig. (17 b)



a

Fig (17)

b

a. Calculate V_{ab}.
b. Determine V_b.
c. Calculate V_c.

Solutions:

a. Voltage divider rule:

$$V_{ab} = \frac{R_1 E}{R_T} = \frac{(2\ \Omega)(10\ \text{V})}{2\ \Omega + 3\ \Omega + 5\ \Omega} = \mathbf{+2\ V}$$

b. Voltage divider rule:

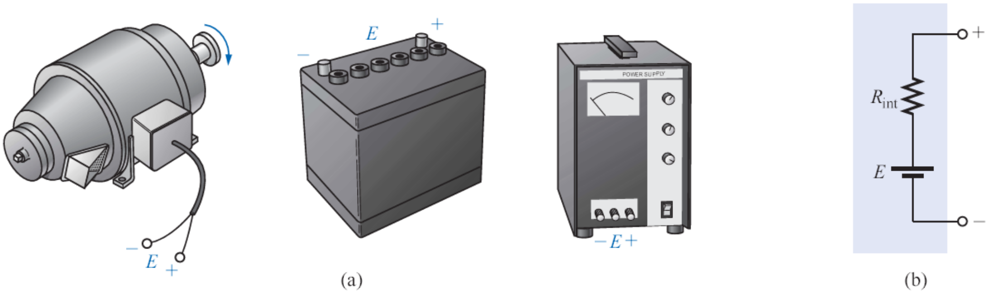
$$V_b = V_{R_2} + V_{R_3} = \frac{(R_2 + R_3)E}{R_T} = \frac{(3\ \Omega + 5\ \Omega)(10\ \text{V})}{10\ \Omega} = \mathbf{8\ V}$$

or $\quad V_b = V_a - V_{ab} = E - V_{ab} = 10\ \text{V} - 2\ \text{V} = \mathbf{8\ V}$

c. $V_c = \text{ground potential} = \mathbf{0\ V}$

INTERNAL RESISTANCE OF VOLTAGE SOURCES

Every source of voltage, whether a generator, battery, or laboratory supply as shown in Fig. 17(a), will have some internal resistance. The equivalent circuit of any source of voltage will therefore appear as shown in Fig. 17(b).



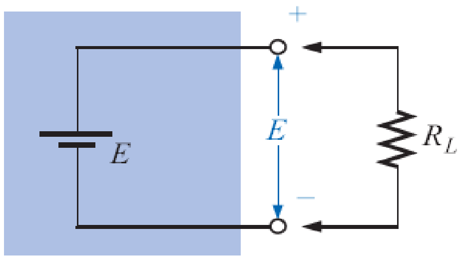
(a) (b)

Fig (17)

(a) Sources of dc voltage; (b) equivalent circuit.

The ideal voltage source has no internal resistance and an output voltage of E volts with

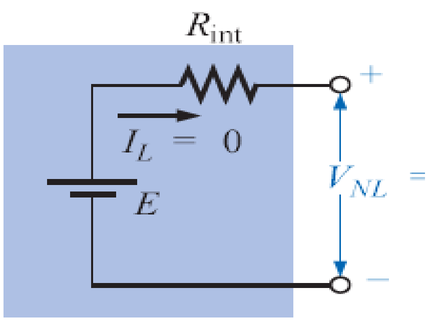
No load or full load. As shown in [fig18 (a)].



(a)

Fig (18)

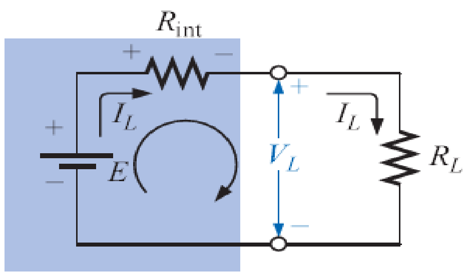
In the practical case [Fig. 18(b)], where we consider the effects of the internal resistance, the output voltage will be E volts only when no-load (I_L = 0) conditions exist.



(b)

Fig (18)

When a load is connected [fig 18c]



(c)

Fig (18)

By applying Kirchhoff's voltage law around the indicated loop of Fig18(c), we obtain:

$$E - I_L R_{int} - V_L = 0$$

or, since $$E = V_{NL}$$

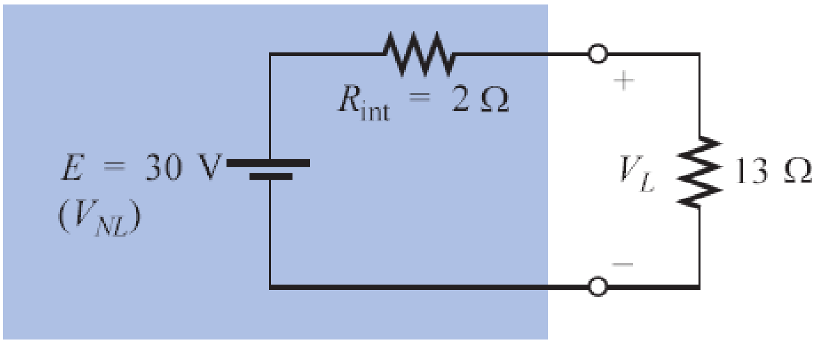
we have $$V_{NL} - I_L R_{int} - V_L = 0$$

and $$\boxed{V_L = V_{NL} - I_L R_{int}}$$

$$R_{int} = \frac{V_{NL} - V_L}{I_L} = \frac{V_{NL}}{I_L} - \frac{I_L R_L}{I_L}$$

and $$\boxed{R_{int} = \frac{V_{NL}}{I_L} - R_L}$$

EXAMPLE 9 The battery of Fig. (19) has an internal resistance of 2 Ω. Find the voltage V_L and the power lost to the internal resistance if the applied load is a 13**Ω** resistor.



Fig(19)

Solution:

$$I_L = \frac{30\text{ V}}{2\,\Omega + 13\,\Omega} = \frac{30\text{ V}}{15\,\Omega} = 2\text{ A}$$
$$V_L = V_{NL} - I_L R_{int} = 30\text{ V} - (2\text{ A})(2\,\Omega)$$
$$P_{lost} = I_L^2 R_{int} = (2\text{ A})^2(2\,\Omega) = (4)(2) = \mathbf{8}$$

Please read and try understand in the first reference Chapter 5.

References:

1- Introductory Circuits Analysis, By Boylested, Tenth (10th) Edition.
2- Schaum's Outline of Theory and Problems of Basic Circuit Analysis, By John O'Malley, Second (2nd) Edition.
3- Any reference that has Direct Current Circuits Analysis (DCCA).

1.1 PARALLEL RESISTANCE

Two elements, branches, or networks are in parallel if they have two points in common.

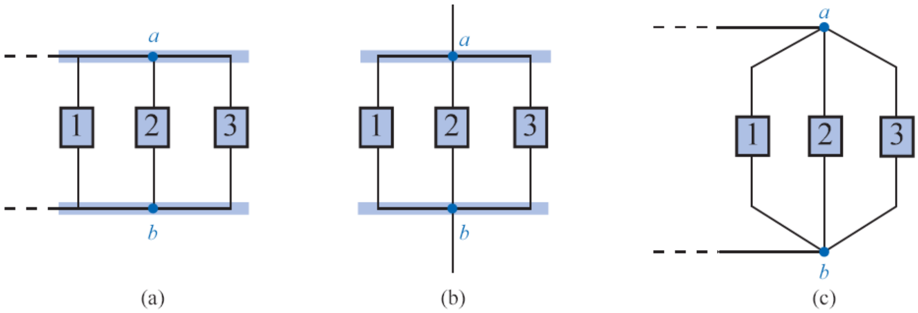


Fig. 1

Fig. 1 Different ways of three parallel elements.

For resistors in parallel as shown in Fig. 2, the total resistance is determined from the following equation:

$$\boxed{\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}}$$

Since G = 1/R, the equation can also be written in terms of conductance levels as follows:

$$\boxed{G_T = G_1 + G_2 + G_3 + \cdots + G_N} \qquad \text{(siemens, S)}$$

In general, however, when the total resistance is desired, the following format is applied:

$$R_T = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \cdots + \dfrac{1}{R_N}}$$

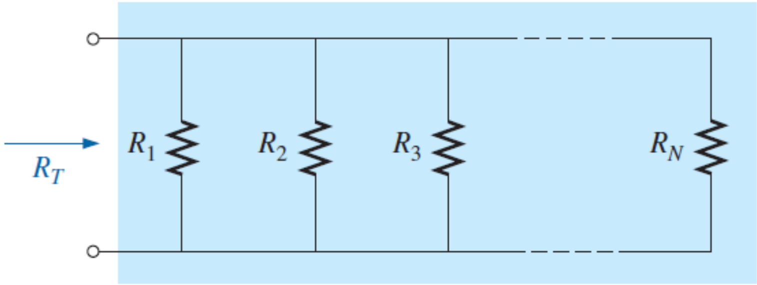


Fig (2) Parallel combination of resistors.

EXAMPLE 1 Find the total resistance of the configuration in Fig. 3.

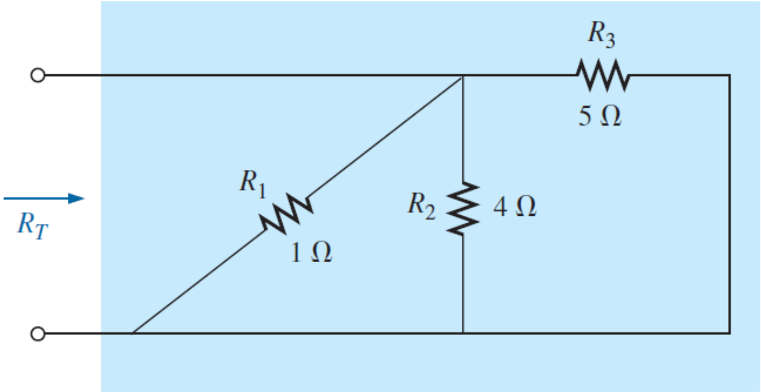


Fig. 3

Solution: First, the network is redrawn as shown in Fig. 4 to clearly demonstrate that all the resistors are in parallel.

$$R_T = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}} = \frac{1}{\dfrac{1}{1\ \Omega} + \dfrac{1}{4\ \Omega} + \dfrac{1}{5\ \Omega}}$$

$$= \frac{1}{1\ \text{S} + 0.25\ \text{S} + 0.2\ \text{S}} = \frac{1}{1.45\ \text{S}} \cong \mathbf{0.69\ \Omega}$$

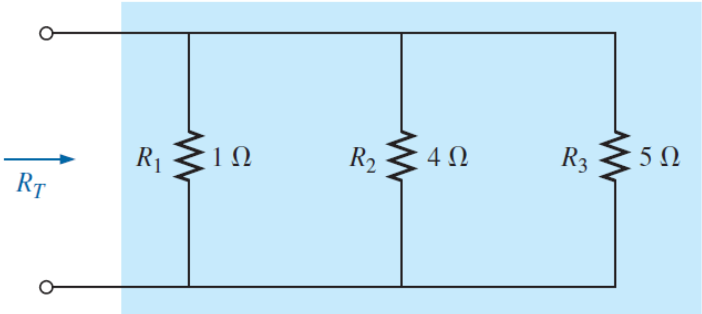


Fig .4 Redrawing the network

Note: **the total resistance of parallel resistors is always less than the value of the smallest resistor.**

For equal resistors in parallel, the equation for the total resistance becomes significantly easier to apply. For *N* equal resistors in parallel, the total resistance becomes:

$$R_T = \frac{1}{\frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \cdots + \frac{1}{R_N}}$$

$$= \frac{1}{N\left(\frac{1}{R}\right)} = \frac{1}{\frac{N}{R}}$$

$$\boxed{R_T = \frac{R}{N}}$$

In other words,
the total resistance of N parallel resistors of equal value is the resistance of one resistor divided by the number (N) of parallel resistors.

EXAMPLE 2 Find the total resistance of the parallel resistors in Fig. 5

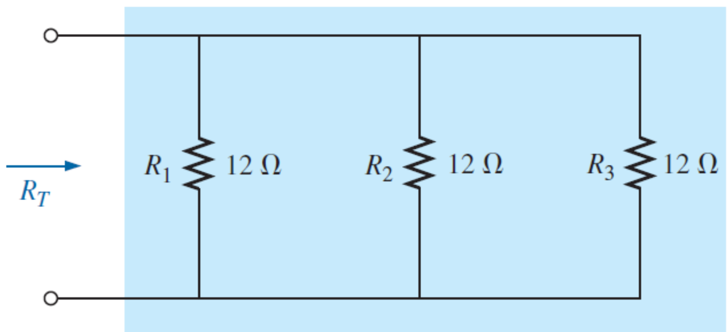


Fig. 5 Three equal parallel resistors to be investigated.

Solution:

$$R_T = \frac{R}{N} = \frac{12\ \Omega}{3} = \mathbf{4\ \Omega}$$

EXAMPLE 3 Find the total resistance for the configuration in Fig. 6.

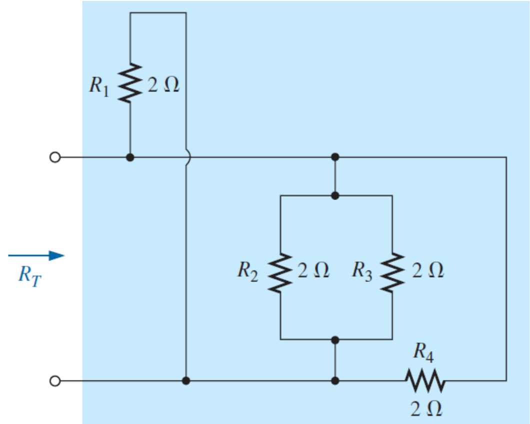


Fig. 6

Solution: Redrawing the network results in the parallel network in Fig. 7.

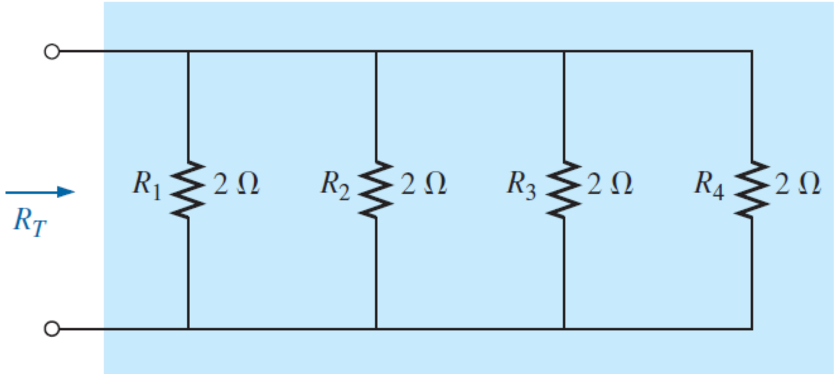


Fig. 7 Network in Fig. 6 redrawn.

$$R_T = \frac{R}{N} = \frac{2\ \Omega}{4} = \mathbf{0.5\ \Omega}$$

Special Case: Two Parallel Resistors

In the vast majority of cases, only two or three parallel resistors will have to be combined. With this in mind, an equation has been derived for two parallel resistors that is easy to apply and removes the need to continually worry about dividing into 1 and possibly misplacing a decimal point. For three parallel resistors, the equation to be derived here can be applied twice, or Equation of Total Resistance can be used.

For two parallel resistors, the total resistance is determined:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

Multiplying the top and bottom of each term of the right side of the equation by the other resistor results in

$$\frac{1}{R_T} = \left(\frac{R_2}{R_2}\right)\frac{1}{R_1} + \left(\frac{R_1}{R_1}\right)\frac{1}{R_2} = \frac{R_2}{R_1R_2} + \frac{R_1}{R_1R_2}$$

$$\frac{1}{R_T} = \frac{R_2 + R_1}{R_1R_2}$$

$$\boxed{R_T = \frac{R_1R_2}{R_1 + R_2}}$$

In words, the equation states that
the total resistance of two parallel resistors is simply the product of their values divided by their sum.

Note:
Recall that series elements can be interchanged without affecting the magnitude of the total resistance. In parallel networks,

parallel resistors can be interchanged without affecting the total resistance.

EXAMPLE 4 Determine the total resistance of the parallel elements in Fig. 8.

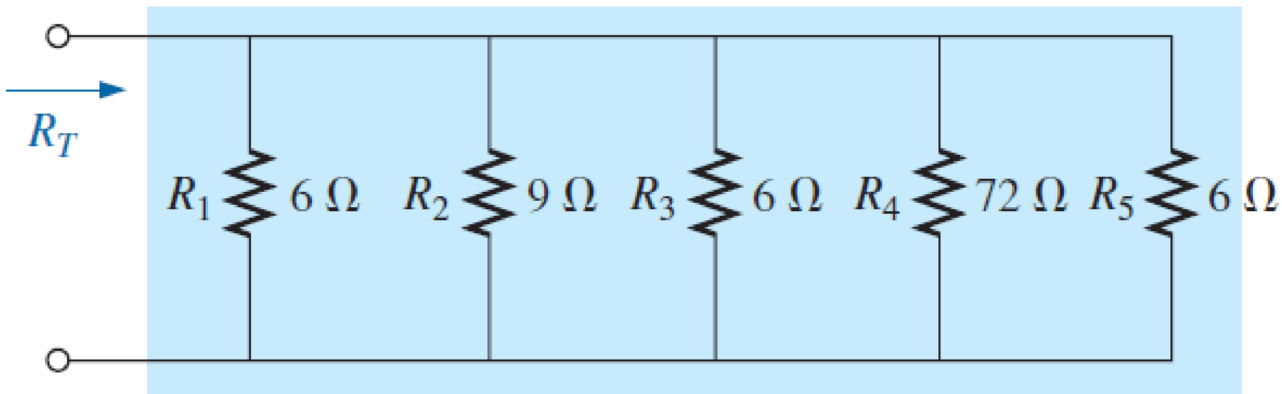


Fig. 8.

Solution: The network is redrawn in Fig. 9.

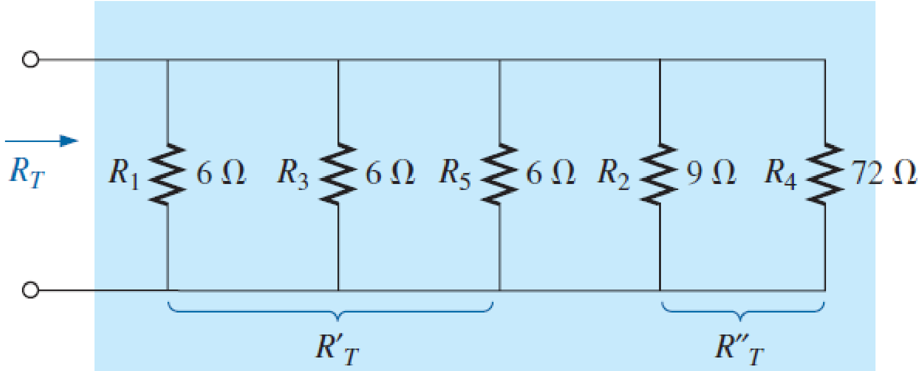


Fig. 9 Redrawing the network

$$R'_T = \frac{R}{N} = \frac{6\ \Omega}{3} = 2\ \Omega$$

$$R''_T = \frac{R_2 R_4}{R_2 + R_4} = \frac{(9\ \Omega)(72\ \Omega)}{9\ \Omega + 72\ \Omega} = \frac{648}{81}\ \Omega = 8\ \Omega$$

$$R_T = \frac{R'_T R''_T}{R'_T + R''_T} = \frac{(2\ \Omega)(8\ \Omega)}{2\ \Omega + 8\ \Omega} = \frac{16}{10}\ \Omega = \mathbf{1.6\ \Omega}$$

EXAMPLE 5 Determine the value of R_2 in Fig. 10 to establish a total resistance of 9 kilo ohms.

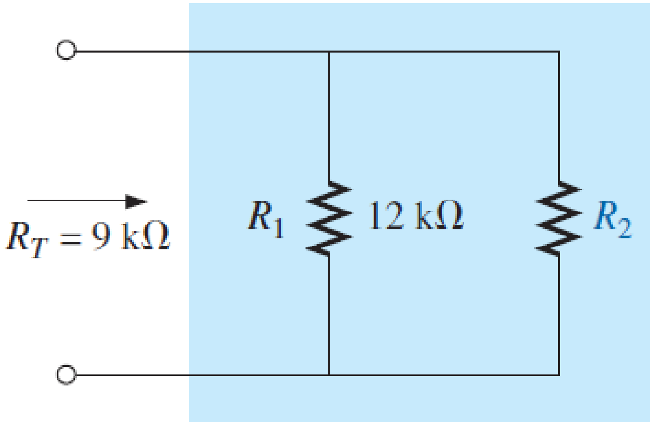


Fig. 10

Solution:

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

$$R_T(R_1 + R_2) = R_1 R_2$$

$$R_T R_1 + R_T R_2 = R_1 R_2$$

$$R_T R_1 = R_1 R_2 - R_T R_2$$

$$R_T R_1 = (R_1 - R_T) R_2$$

and

$$R_2 = \frac{R_T R_1}{R_1 - R_T}$$

Substituting values:

$$R_2 = \frac{(9\ \text{k}\Omega)(12\ \text{k}\Omega)}{12\ \text{k}\Omega - 9\ \text{k}\Omega} = \frac{108}{3}\ \text{k}\Omega = \mathbf{36\ k\Omega}$$

EXAMPLE 6 Determine the values of R_1, R_2 and R_3 in Fig. 11 if $R_2 = 2R_1$, $R_3 = 2R_2$, and the total resistance is 16 kilo ohms.

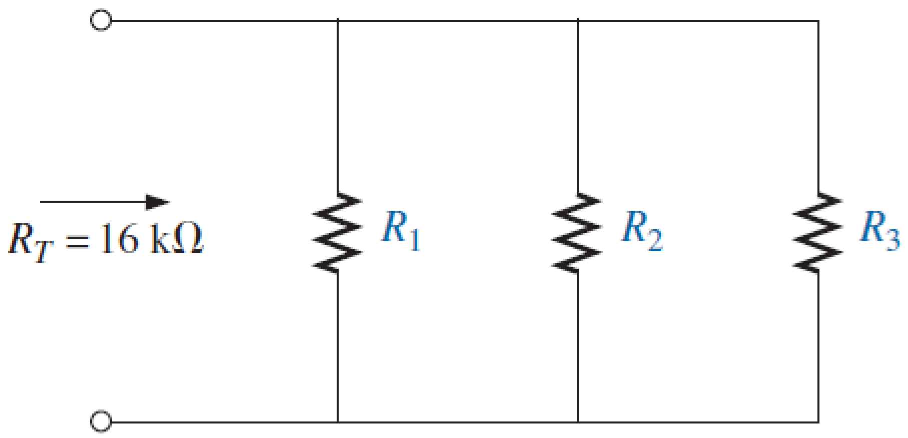


Fig. 11

Solution:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

However, $R_2 = 2R_1$ and $R_3 = 2R_2 = 2(2R_1) = 4R_1$

so that
$$\frac{1}{16\ \text{k}\Omega} = \frac{1}{R_1} + \frac{1}{2R_1} + \frac{1}{4R_1}$$

and
$$\frac{1}{16\ \text{k}\Omega} = \frac{1}{R_1} + \frac{1}{2}\left(\frac{1}{R_1}\right) + \frac{1}{4}\left(\frac{1}{R_1}\right)$$

or
$$\frac{1}{16\ \text{k}\Omega} = 1.75\left(\frac{1}{R_1}\right)$$

resulting in $R_1 = 1.75(16\ \text{k}\Omega) = \mathbf{28\ k\Omega}$

so that $R_2 = 2R_1 = 2(28\ \text{k}\Omega) = \mathbf{56\ k\Omega}$

and $R_3 = 2R_2 = 2(56\ \text{k}\Omega) = \mathbf{112\ k\Omega}$

1.2 PARALLEL CIRCUITS

A **parallel circuit** can now be established by connecting a supply across a set of parallel resistors as shown in Fig.12. The positive terminal of the supply is directly connected to the top of each resistor, while the negative terminal is connected to the bottom of each resistor. Therefore, it should be quite clear that the applied voltage is the same across each resistor.

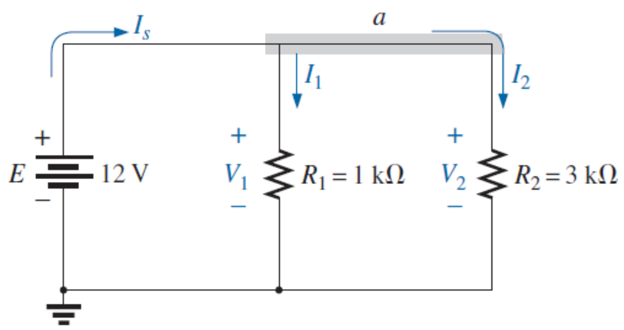


Fig. 12

In general,

the voltage is always the same across parallel elements.

For the voltages of the circuit in Fig. 12, the result is that

$$\boxed{V_1 = V_2 = E}$$

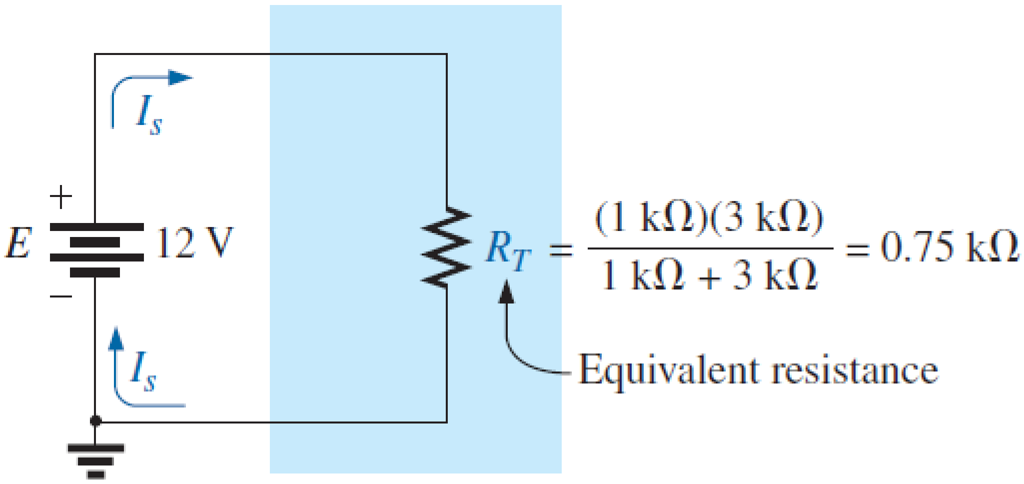


Fig. 13

Replacing the parallel resistors in Fig. 13 with the equivalent total resistance.

The source current can then be determined using Ohm's law:

$$\boxed{I_s = \frac{E}{R_T}}$$

Since the voltage is the same across parallel elements, the current through each resistor can also be determined using Ohm's law. That is,

$$\boxed{I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} \quad \text{and} \quad I_2 = \frac{V_2}{R_2} = \frac{E}{R_2}}$$

The direction for the currents is dictated by the polarity of the voltage across the resistors. Recall that for a resistor, current enters the positive side of a potential drop and leaves the negative. The result, as shown in Fig. 12, is that the source current enters point a, and currents I_1 and I_2 leave the same point.

An excellent analogy for describing the flow of charge through the network of Fig. 12 is the flow of water through the parallel pipes of Fig. 14.

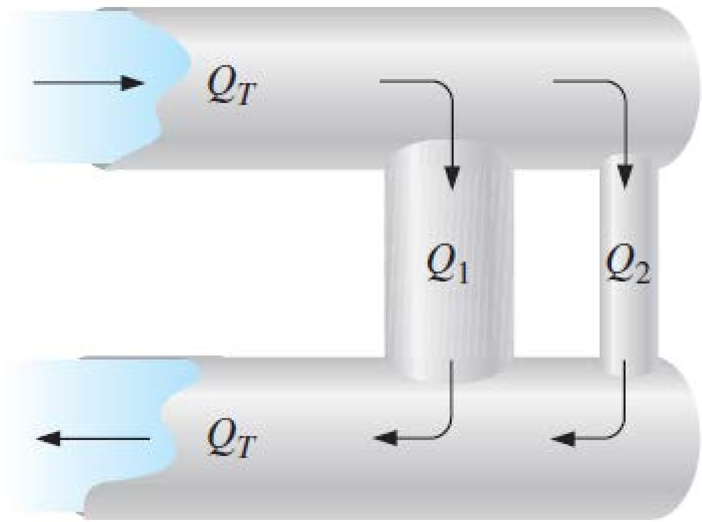


Fig. 14

The larger pipe with less "resistance" to the flow of water will have a larger flow of water through it. The thinner pipe with its increased "resistance" level will have less water through it. In any case, the total water entering the pipes at the top Q_T must equal that leaving at the bottom, with $Q_T = Q_1 + Q_2$.

The relationship between the source current and the parallel resistor currents can be derived by simply taking the equation for the total resistance in equation of total resistance.

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

Multiplying both sides by the applied voltage:

$$E\left(\frac{1}{R_T}\right) = E\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

resulting in

$$\frac{E}{R_T} = \frac{E}{R_1} + \frac{E}{R_2}$$

Then note that $E/R_1 = I_1$ and $E/R_2 = I_2$ to obtain

$$\boxed{I_s = I_1 + I_2}$$

The result reveals a very important property of parallel circuits:

For single-source parallel networks, the source current (I_s) is always equal to the sum of the individual branch currents.

The duality that exists between series and parallel circuits continues to surface as we proceed through the basic equations for electric circuits.

for a parallel circuit, the source current equals the sum of the branch currents, while for a series circuit, the applied voltage equals the sum of the voltage drops.

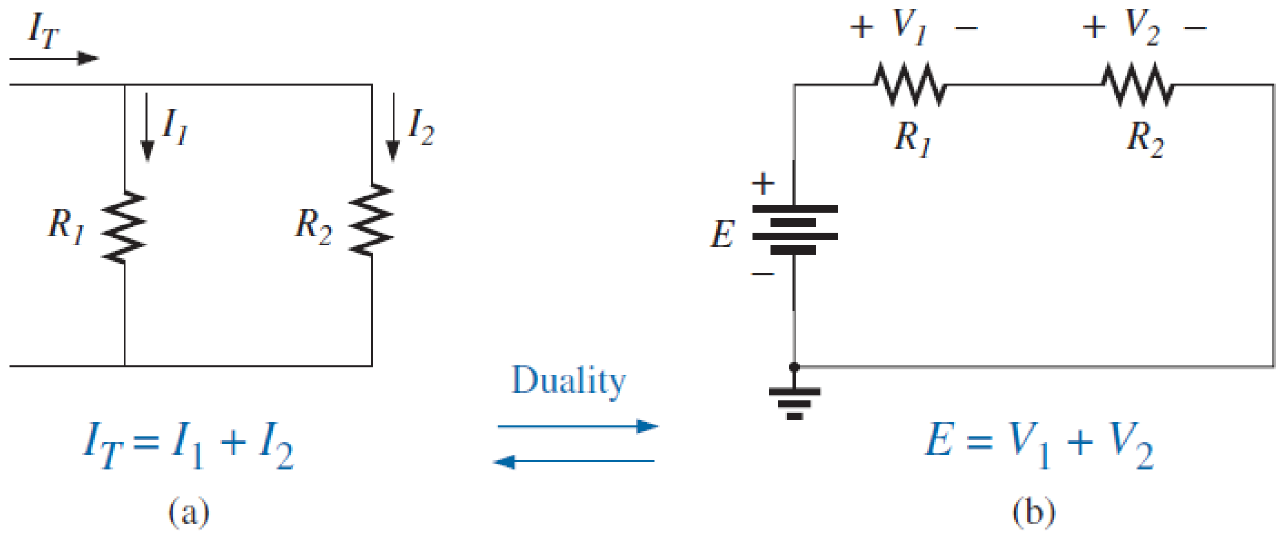


Fig. 15 Demonstrating the duality that exists between series and parallel circuits.

EXAMPLE 7 For the parallel network in Fig. 16.

a. Find the total resistance.

b. Calculate the source current.

c. Determine the current through each branch.

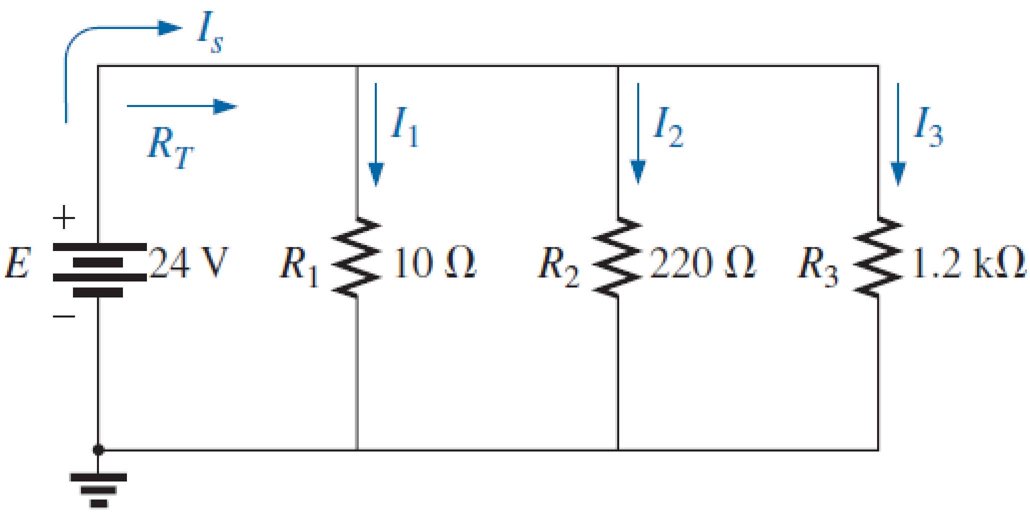


Fig. 16

Solutions:

a.

$$R_T = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \frac{1}{\frac{1}{10\ \Omega}+\frac{1}{220\ \Omega}+\frac{1}{1.2\ \text{k}\Omega}}$$

$$= \frac{1}{100\times10^{-3}+4.545\times10^{-3}+0.833\times10^{-3}} = \frac{1}{105.38\times10^{-3}}$$

$$R_T = \mathbf{9.49\ \Omega}$$

Note that the total resistance is less than the smallest parallel resistor.

b. Using Ohm's law:

$$I_s = \frac{E}{R_T} = \frac{24\ \text{V}}{9.49\ \Omega} = \mathbf{2.53\ A}$$

c. Applying Ohm's law:

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{24\ \text{V}}{10\ \Omega} = \mathbf{2.4\ A}$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{24\ \text{V}}{220\ \Omega} = \mathbf{0.11\ A}$$

$$I_3 = \frac{V_3}{R_3} = \frac{E}{R_3} = \frac{24\ \text{V}}{1.2\ \text{k}\Omega} = \mathbf{0.02\ A}$$

for parallel resistors, the greatest current will exist in the branch with the least resistance.

A more powerful statement is that
current always seeks the path of least resistance.

1.3 POWER DISTRIBUTION IN A PARALLEL CIRCUIT

for any network composed of resistive elements, the power applied by the battery will equal that dissipated by the resistive elements.

For the parallel circuit in Fig. 17:

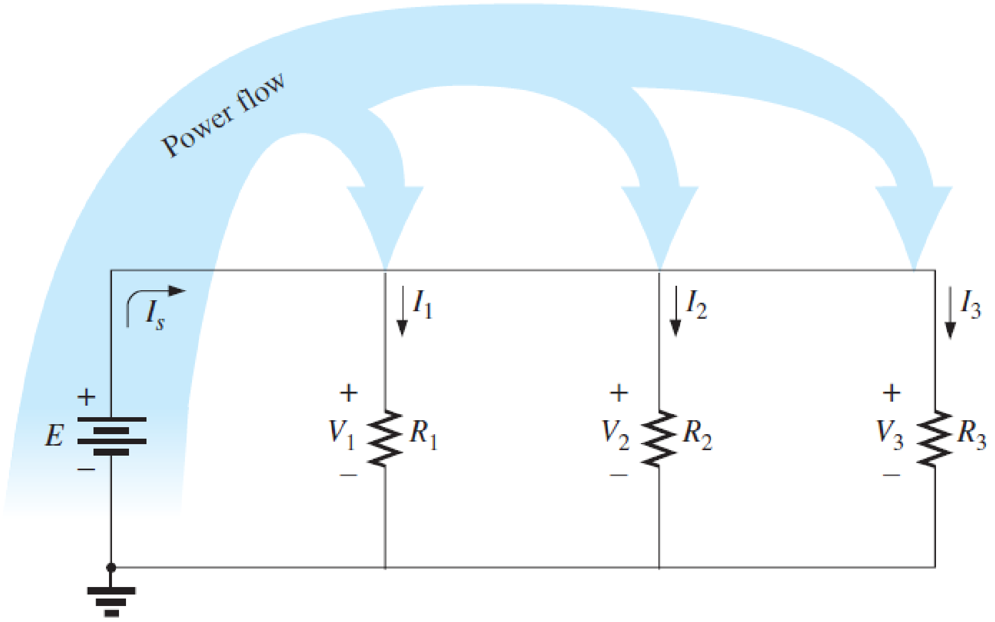


Fig. 17
Power flow in a dc parallel network.

$$\boxed{P_E = P_{R_1} + P_{R_2} + P_{R_3}}$$

which is exactly the same as obtained for the series combination.

The power delivered by the source in the same:

$$\boxed{P_E = EI_s} \qquad \text{(watts, W)}$$

as is the equation for the power to each resistor.

$$\boxed{P_1 = V_1 I_1 = I_1^2 R_1 = \frac{V_1^2}{R_1}} \qquad \text{(watts, W)}$$

in a parallel resistive network, the larger the resistor, the less the power absorbed.

EXAMPLE 8 For the parallel network in Fig. 18
a. Determine the total resistance R_T.
b. Find the source current and the current through each resistor.
c. Calculate the power delivered by the source.
d. Determine the power absorbed by each parallel resistor.
e. Verify the power applied by the battery will equal that dissipated by the resistive elements.

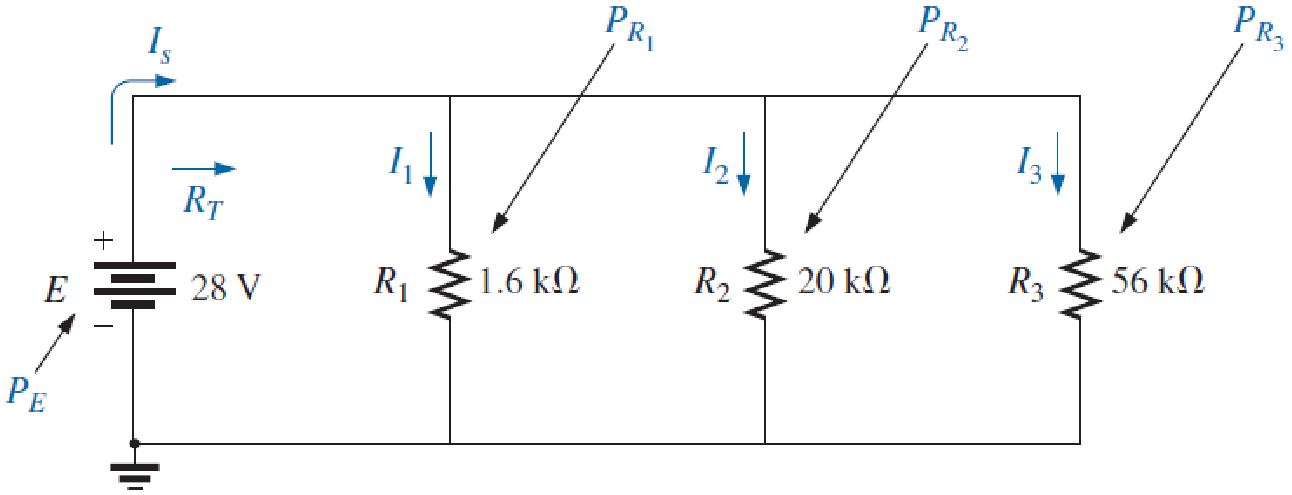


Fig. 18

Solutions:

a.

$$R_T = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \frac{1}{\frac{1}{1.6\ \text{k}\Omega}+\frac{1}{20\ \text{k}\Omega}+\frac{1}{56\ \text{k}\Omega}}$$

$$= \frac{1}{625\times10^{-6}+50\times10^{-6}+17.867\times10^{-6}} = \frac{1}{692.867\times10^{-6}}$$

and $R_T = \mathbf{1.44\ k\Omega}$

b. Applying Ohm's law:

$$I_s = \frac{E}{R_T} = \frac{28\ \text{V}}{1.44\ \text{k}\Omega} = \mathbf{19.44\ mA}$$

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{28\ \text{V}}{1.6\ \text{k}\Omega} = \mathbf{17.5\ mA}$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{28\ \text{V}}{20\ \text{k}\Omega} = \mathbf{1.4\ mA}$$

$$I_3 = \frac{V_3}{R_3} = \frac{E}{R_3} = \frac{28\ \text{V}}{56\ \text{k}\Omega} = \mathbf{0.5\ mA}$$

c.

$$P_E = EI_s = (28\text{ V})\,(19.4\text{ mA}) = \mathbf{543.2\text{ mW}}$$

d. Applying each form of the power equation:

$$P_1 = V_1I_1 = EI_1 = (28\text{ V})(17.5\text{ mA}) = \mathbf{490\text{ mW}}$$

$$P_2 = I_2^2R_2 = (1.4\text{ mA})^2(20\text{ k}\Omega) = \mathbf{39.2\text{ mW}}$$

$$P_3 = \frac{V_3^2}{R_3} = \frac{E^2}{R_3} = \frac{(28\text{ V})^2}{56\text{ k}\Omega} = \mathbf{14\text{ mW}}$$

A review of the results clearly substantiates the fact that the larger the resistor, the less the power absorbed.

e.

$$P_E = P_{R_1} + P_{R_2} + P_{R_3}$$

$$\mathbf{543.2\text{ mW}} = 490\text{ mW} + 39.2\text{ mW} + 14\text{ mW} = \mathbf{543.2\text{ mW}} \quad \text{(checks)}$$

1.4 KIRCHHOFF'S CURRENT LAW (KCL)

The algebraic sum of the currents entering and leaving a junction (or region) of a network is zero.

The sum of the currents entering a junction (or region) of a network must equal the sum of the currents leaving the same junction (or region).

In equation form, the above statement can be written as follows:

$$\boxed{\Sigma I_i = \Sigma I_o}$$

with I_i representing the current entering, or "in," and I_o representing the current leaving, or "out."

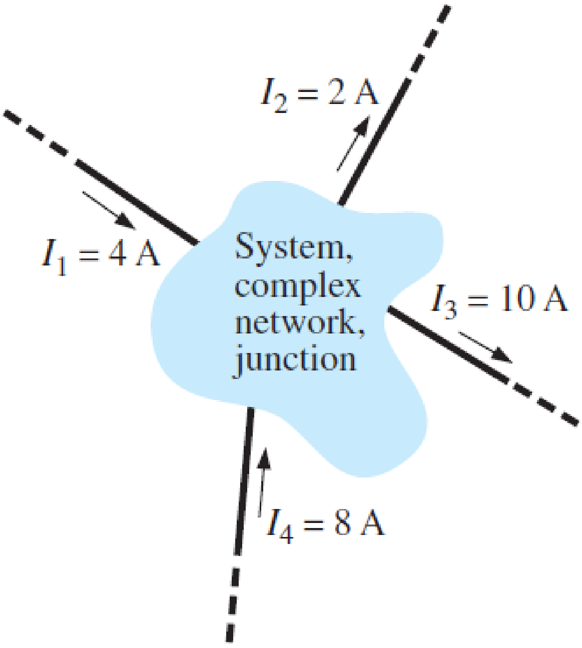


Fig. 19

$$\Sigma I_i = \Sigma I_o$$
$$I_1 + I_4 = I_2 + I_3$$
$$4\text{ A} + 8\text{ A} = 2\text{ A} + 10\text{ A}$$
$$\mathbf{12\text{ A} = 12\text{ A}} \quad \text{(checks)}$$

In technology, the term **node** is commonly used to refer to a junction of two or more branches. Therefore, this term is used frequently in the analyses to follow.

EXAMPLE 9 Determine currents I_3 and I_4 in Fig. 20 using Kirchhoff's current law.

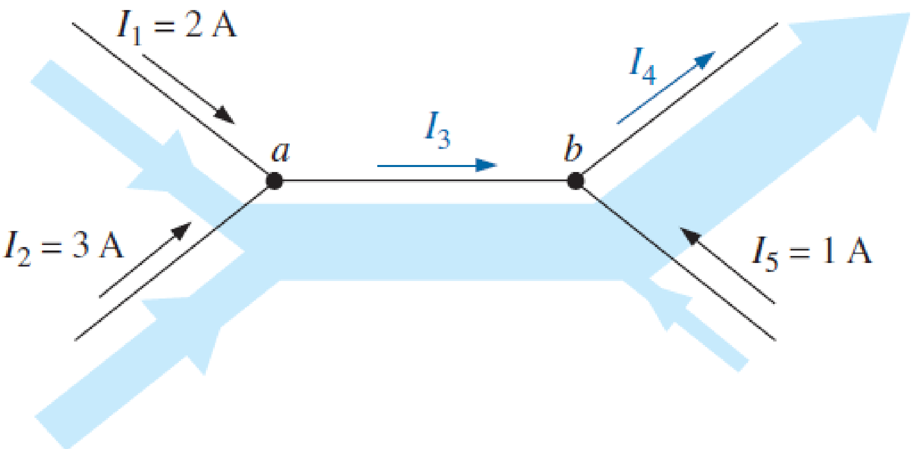


Fig. 20

Solution: There are two junctions or nodes in Fig. 20. Node *a* has only one unknown, while node *b* has two unknowns. Since a single equation can be used to solve for only one unknown, we must apply Kirchhoff's current law to node *a* first.

At node *a:*

$$\Sigma I_i = \Sigma I_o$$
$$I_1 + I_2 = I_3$$
$$2\text{ A} + 3\text{ A} = I_3 = \mathbf{5\text{ A}}$$

At node *b*, using the result just obtained:

$$\Sigma I_i = \Sigma I_o$$
$$I_3 + I_5 = I_4$$
$$5\text{ A} + 1\text{ A} = I_4 = \mathbf{6\text{ A}}$$

Note that in Fig. 20, the width of the blue shaded regions matches the magnitude of the current in that region.

EXAMPLE 10 Determine currents I_1, I_3, I_4, and I_5 for the network in Fig. 21.

Solution: In this configuration, four nodes are defined. Nodes *a* and *c* have only one unknown current at the junction, so Kirchhoff's current law can be applied at either junction.

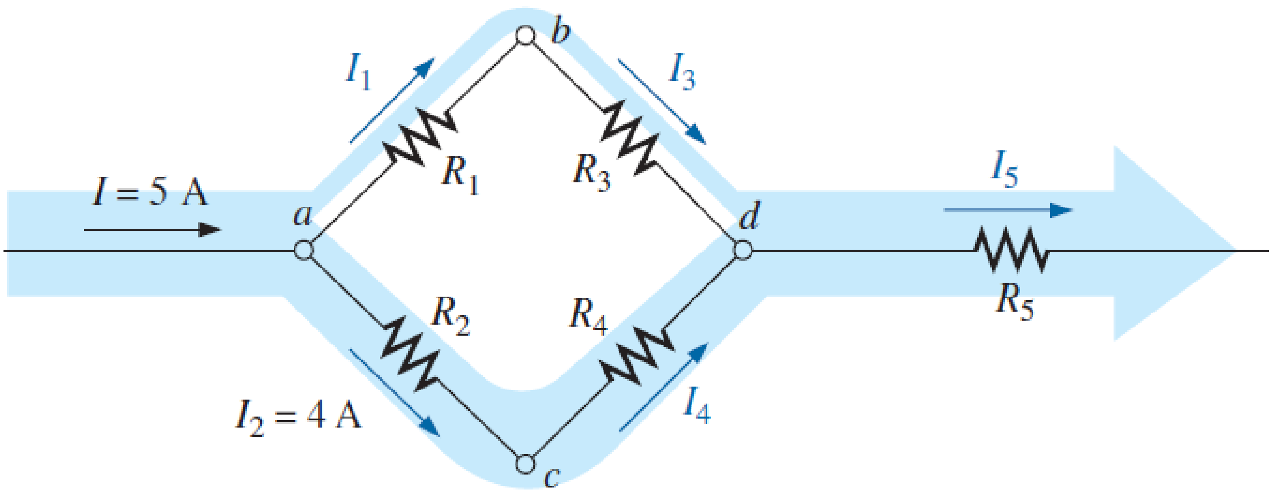


Fig. 21 Four-node configuration for the Example.

At node *a:*

$$\Sigma I_i = \Sigma I_o$$
$$I = I_1 + I_2$$
$$5\text{ A} = I_1 + 4\text{ A}$$
$$I_1 = 5\text{ A} - 4\text{ A} = \mathbf{1\ A}$$

At node *c:*

$$\Sigma I_i = \Sigma I_o$$
$$I_2 = I_4$$
$$I_4 = I_2 = \mathbf{4\ A}$$

Using the above results at the other junctions results in the following.

At node *b:*

$$\Sigma I_i = \Sigma I_o$$
$$I_1 = I_3$$
$$I_3 = I_1 = \mathbf{1\ A}$$

At node *d:*

$$\Sigma I_i = \Sigma I_o$$
$$I_3 + I_4 = I_5$$
$$1\text{ A} + 4\text{ A} = I_5 = \mathbf{5\ A}$$

1.5 CURRENT DIVIDER RULE (CDR)

For series circuits we have the powerful voltage divider rule for finding the voltage across a resistor in a series circuit. We now introduce the equally powerful **current divider rule (CDR)** for finding the current through a resistor in a parallel circuit.

For two parallel elements of equal value, the current will divide equally. For parallel elements with different values, the smaller the resistance, the greater the share of input current. For parallel elements of different values, the current will split with a ratio equal to the inverse of their resistor values.

EXAMPLE 11

a) Determine currents I_1 and I_3 for the network in Fig. 22.
b) Find the source current I_s.

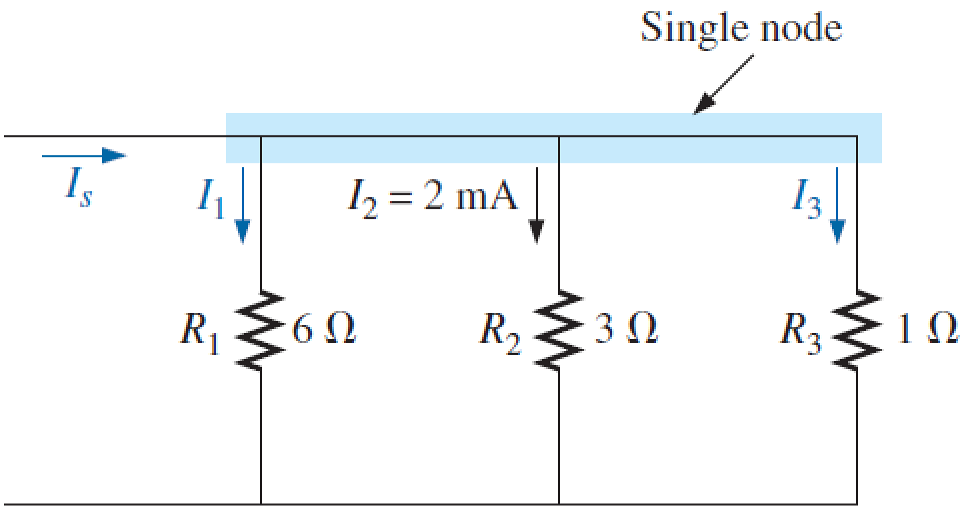


Fig. 22

Solutions:

a. Since R_1 is twice R_2, the current I_1 must be one-half I_2, and

$$I_1 = \frac{I_2}{2} = \frac{2 \text{ mA}}{2} = \mathbf{1\ mA}$$

Since R_2 is three times R_3, the current I_3 must be three times I_2, and

$$I_3 = 3I_2 = 3(2 \text{ mA}) = \mathbf{6\ mA}$$

b. Applying Kirchhoff's current law:

$$\Sigma I_i = \Sigma I_o$$
$$I_s = I_1 + I_2 + I_3$$
$$I_s = 1 \text{ mA} + 2 \text{ mA} + 6 \text{ mA} = \mathbf{9\ mA}$$

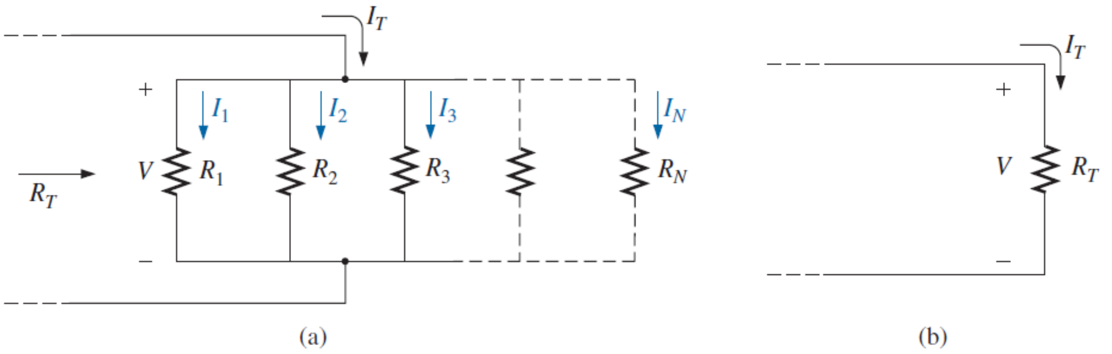


Fig. 23 Deriving the current divider rule: (a) parallel network of N parallel resistors; (b) reduced equivalent of part (a).

Although the above discussions and examples allowed us to determine the relative magnitude of a current based on a known level, they do not provide the magnitude of a current through a branch of a parallel network if only the total entering current is known. The result is a need for the current divider rule which will be derived using the parallel configuration in Fig. 23(a). The current I_T (using the subscript T to indicate the total entering current) splits between the N parallel resistors and then gathers itself together again at the bottom of the configuration.

In Fig. 23(b), the parallel combination of resistors has been replaced by a single resistor equal to the total resistance of the parallel combination as determined in the previous sections.

The current I_T can then be determined using Ohm's law:

$$I_T = \frac{V}{R_T}$$

Since the voltage V is the same across parallel elements, the following is true:

$$V = I_1R_1 = I_2R_2 = I_3R_3 = \cdots = I_xR_x$$

where the product I_xR_x refers to any combination in the series. Substituting for V in the above equation for I_T, we have

$$I_T = \frac{I_xR_x}{R_T}$$

Solving for I_x , the final result is the **current divider rule:**

$$\boxed{I_x = \frac{R_T}{R_x}I_T}$$

which states that
the current through any branch of a parallel resistive network is equal to the total resistance of the parallel network divided by the resistor of interest and multiplied by the total current entering the parallel configuration.
Since R_T and I_T are constants, for a particular configuration the larger the value of R_x (in the denominator), the smaller the value of I_x for that branch, confirming the fact that current always seeks the path of least resistance.

EXAMPLE 12 For the parallel network in Fig. 24, determine current I_1 using Eq. CDR.

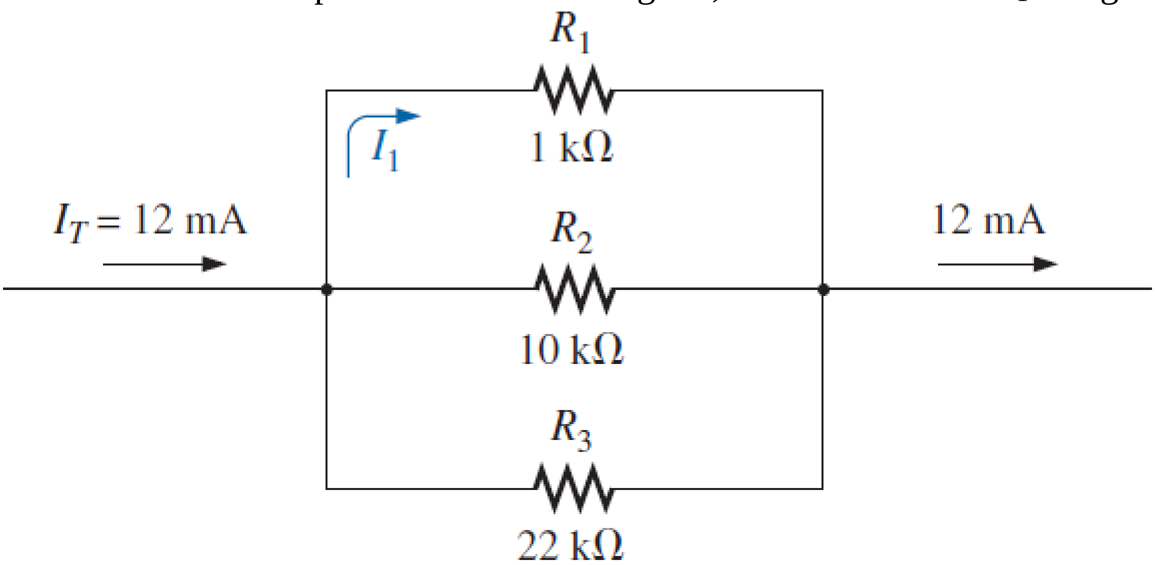


Fig. 24 Using the current divider rule to calculate current I_1

Solution:

$$R_T = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}}$$

$$= \frac{1}{\frac{1}{1\text{ k}\Omega}+\frac{1}{10\text{ k}\Omega}+\frac{1}{22\text{ k}\Omega}}$$

$$= \frac{1}{1\times10^{-3}+100\times10^{-6}+45.46\times10^{-6}}$$

$$= \frac{1}{1.145\times10^{-3}} = \mathbf{873.01\ \Omega}$$

$$I_1 = \frac{R_T}{R_1}I_T$$

$$= \frac{(873.01\ \Omega)}{1\text{ k}\Omega}(12\text{ mA}) = (0.873)(12\text{ mA}) = \mathbf{10.48\ mA}$$

and the smallest parallel resistor receives the majority of the current.

Special Case: Two Parallel Resistors

For the case of two parallel resistors as shown in Fig 25, the total resistance is determined by

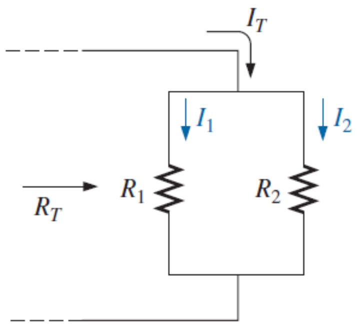


Fig. 25 Deriving the current divider rule for the special case of only two parallel resistors.

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

Substituting R_T into Eq. CDR for current I_1 results in

$$I_1 = \frac{R_T}{R_1} I_T = \frac{\left(\dfrac{R_1 R_2}{R_1 + R_2}\right)}{R_1} I_T$$

And

$$\boxed{I_1 = \left(\frac{R_2}{R_1 + R_2}\right) I_T}$$

Similarly, for I_2,

$$\boxed{I_2 = \left(\frac{R_1}{R_1 + R_2}\right) I_T}$$

Equation I_1, I_2 states that

for two parallel resistors, the current through one is equal to the other resistor times the total entering current divided by the sum of the two resistors.

Since the combination of two parallel resistors is probably the most common parallel configuration, the simplicity of the format for Equation (I_1, I_2) suggests that it is worth memorizing. Take particular note, however, that the denominator of the equation is simply the *sum*, not the total resistance, of the combination.

EXAMPLE 13 Determine current I_2 for the network in Fig. 26 using the current divider rule.

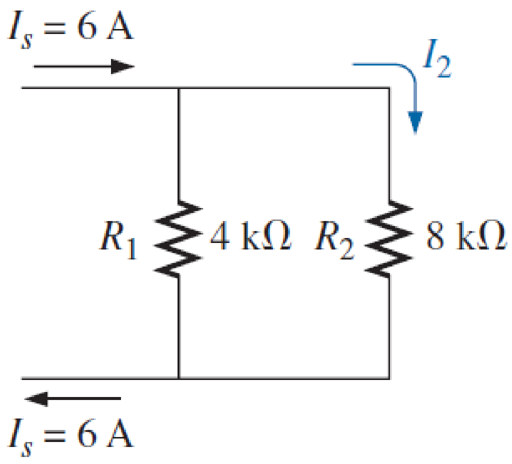


Fig. 26

Solution:

$$I_2 = \left(\frac{R_1}{R_1 + R_2}\right) I_T$$

$$= \left(\frac{4\text{ k}\Omega}{4\text{ k}\Omega + 8\text{ k}\Omega}\right) 6\text{ A} = (0.333)(6\text{ A}) = \mathbf{2\text{ A}}$$

$$I_2 = \frac{R_T}{R_2} I_T$$

with $$R_T = 4\text{ k}\Omega \parallel 8\text{ k}\Omega = \frac{(4\text{ k}\Omega)(8\text{ k}\Omega)}{4\text{ k}\Omega + 8\text{ k}\Omega} = 2.667\text{ k}\Omega$$

and $$I_2 = \left(\frac{2.667\text{ k}\Omega}{8\text{ k}\Omega}\right) 6\text{ A} = (0.333)(6\text{ A}) = \mathbf{2\text{ A}}$$

matching the above solution.

EXAMPLE 14 Determine resistor R_1 in Fig. 27 to implement the division of current shown.

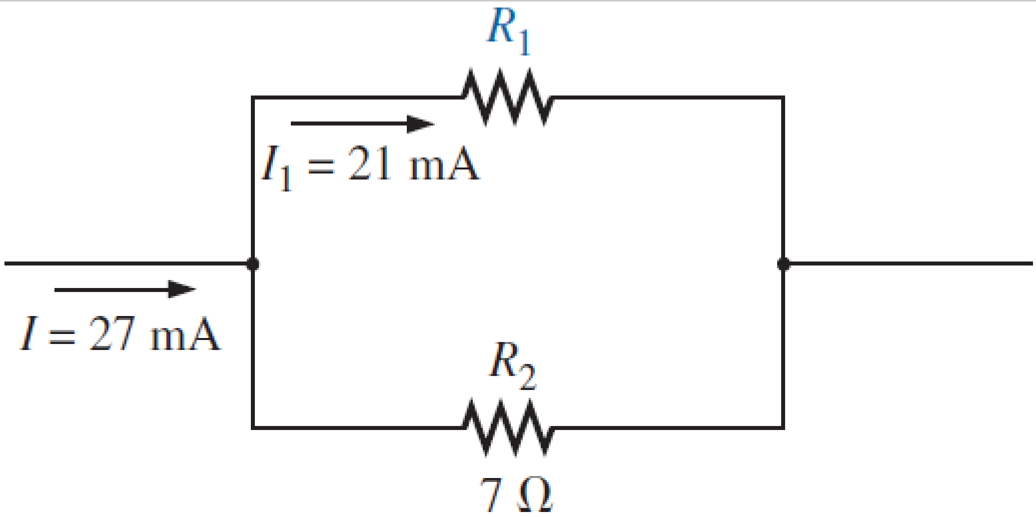


Fig. 27 A design-type problem for two parallel resistors.

Solution: There are essentially two approaches to this type of problem. One involves the direct substitution of known values into the current divider rule equation followed by a mathematical analysis. The other is the sequential application of the basic laws of electric circuits.

First we will use the latter approach.

Applying Kirchhoff's current law:

$$\Sigma I_i = \Sigma I_o$$

$$I = I_1 + I_2$$

$$27 \text{ mA} = 21 \text{ mA} + I_2$$

and $$I_2 = 27 \text{ mA} - 21 \text{ mA} = 6 \text{ mA}$$

The voltage V_2: $$V_2 = I_2R_2 = (6 \text{ mA})(7\ \Omega) = 42 \text{ mV}$$

so that $$V_1 = V_2 = 42 \text{ mV}$$

Finally, $$R_1 = \frac{V_1}{I_1} = \frac{42 \text{ mV}}{21 \text{ mA}} = \mathbf{2\ \Omega}$$

Now for the other approach using the current divider rule:

$$I_1 = \frac{R_2}{R_1 + R_2} I_T$$

$$21 \text{ mA} = \left(\frac{7\ \Omega}{R_1 + 7\ \Omega}\right) 27 \text{ mA}$$

$$(R_1 + 7\ \Omega)(21 \text{ mA}) = (7\ \Omega)(27 \text{ mA})$$

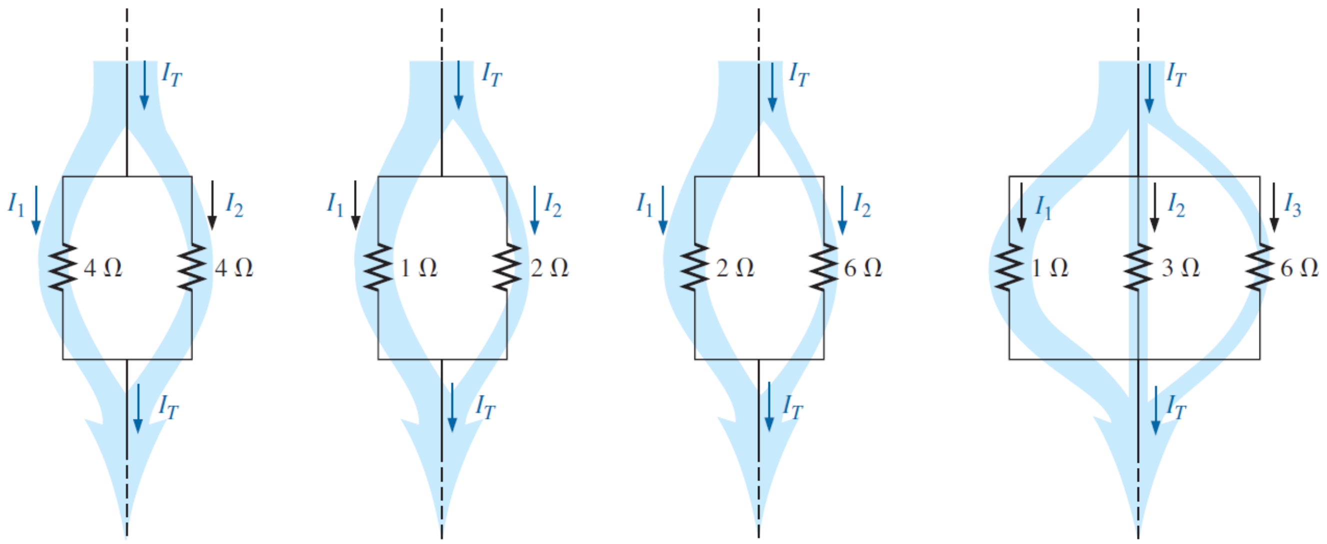
$$(21 \text{ mA})R_1 + 147 \text{ mV} = 189 \text{ mV}$$

$$(21 \text{ mA})R_1 = 189 \text{ mV} - 147 \text{ mV} = 42 \text{ mV}$$

and

$$R_1 = \frac{42 \text{ mV}}{21 \text{ mA}} = \mathbf{2\ \Omega}$$

In summary, therefore, remember that current always seeks the path of least resistance, and the ratio of the resistor values is the inverse of the resulting current levels, as shown in Fig 28. The thickness of the blue bands in Fig. 28 reflects the relative magnitude of the current in each branch.



$$I_1 = I_2 = \frac{I_T}{2}$$

$$I_1 = 2I_2$$

$$I_1 = \left(\frac{6}{2}\right) I_2 = 3I_2$$

$$I_1 = 6I_3$$
$$I_1 = 3I_2$$
$$I_2 = \left(\frac{6}{3}\right) I_3 = 2I_3$$

Fig. 28 Demonstrating how current divides through equal and unequal parallel resistors.

1.6 OPEN AND SHORT CIRCUITS

An **open circuit** is two isolated terminals not connected by an element of any kind, as shown in Fig. 29(a).

Since a path for conduction does not exist, the current associated with an open circuit must always be **zero**. The voltage across the open circuit, however, can be any value, as determined by the system it is connected to. In summary, therefore,
an open circuit can have a potential difference (voltage) across its terminals, but the current is always zero amperes.

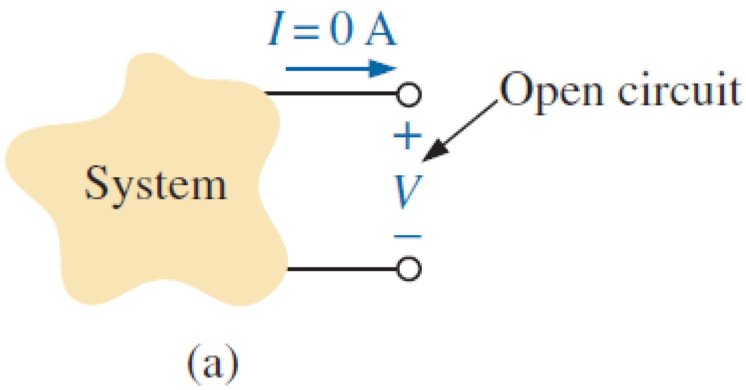


Fig. 29-a

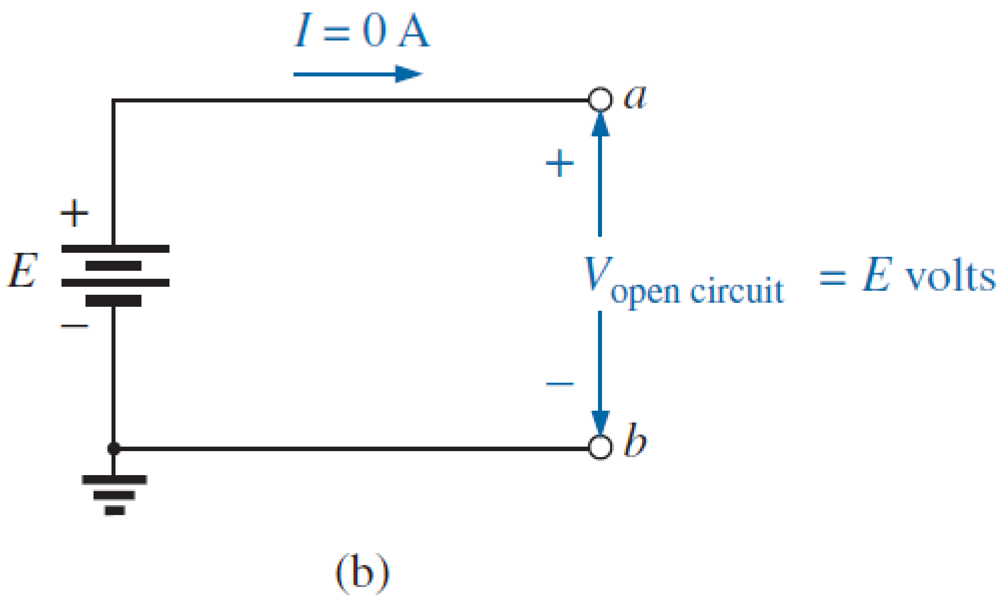


Fig. 29-b

In Fig. 29(b), an open circuit exists between terminals *a* and *b*. The voltage across the open-circuit terminals is the **supply voltage**, but the current is **zero** due to the absence of a complete circuit.

A **short circuit** is a very low resistance, direct connection between two terminals of a network, as shown in Fig. 30. The current through the short circuit can be any value, as determined by the system it is connected to, but the voltage across the short circuit is always zero volts because the resistance of the short circuit is assumed to be essentially zero ohms and

$$V = IR = I(0\ \Omega) = 0\ \text{V}.$$

In summary, therefore,

a short circuit can carry a current of a level determined by the external circuit, but the potential difference (voltage) across its terminals is always zero volts.

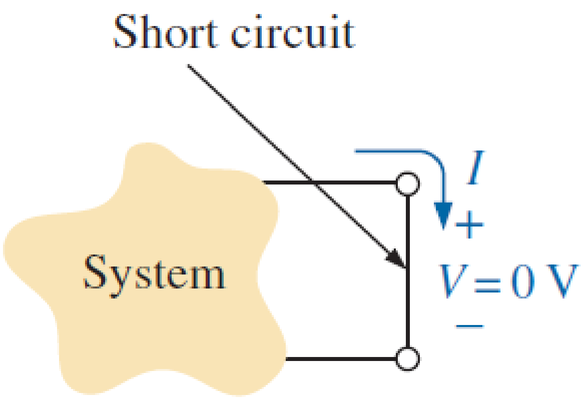


Fig. 30

EXAMPLE 15 Determine voltage V_{ab} for the network in Fig. 31.

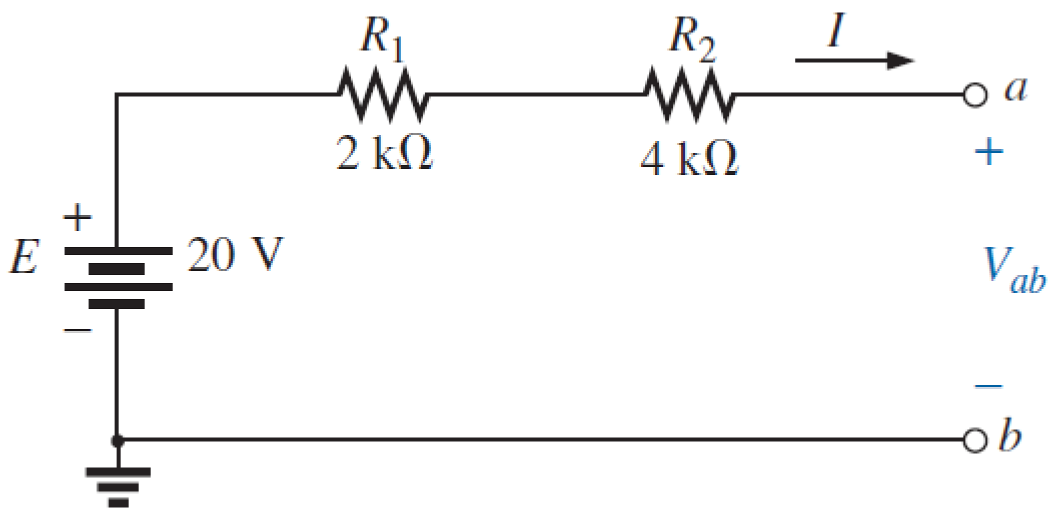


Fig. 31

Solution: The open circuit requires that I be zero amperes.

The voltage drop across both resistors is therefore zero volts since

$$V = IR = (0)R = 0\ \text{V}.$$

Applying Kirchhoff's voltage law around the closed loop,

$$V_{ab} = E = \mathbf{20\ V}$$

EXAMPLE 16 Determine voltages V_{ab} and V_{cd} for the network in Fig. 32.

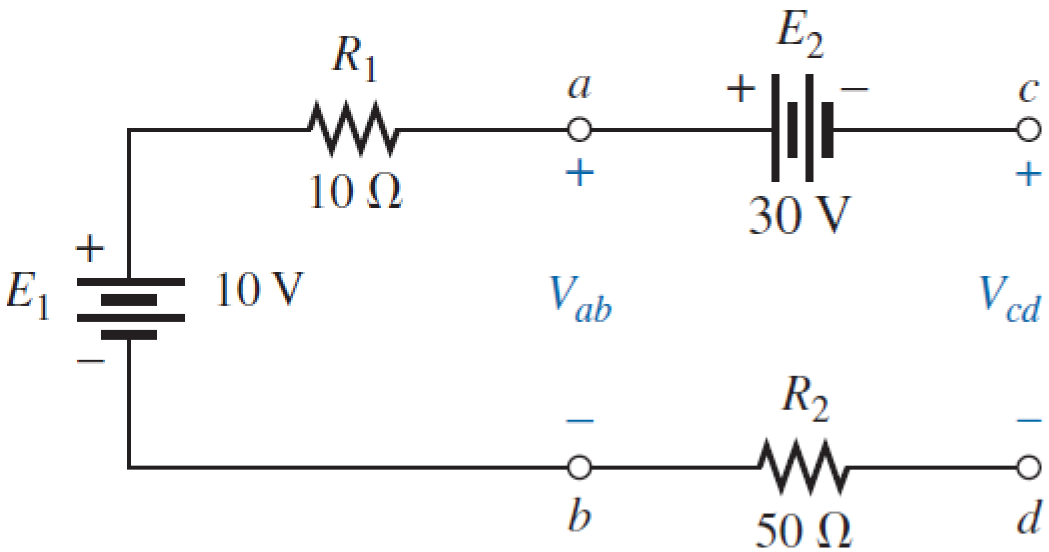


Fig. 32

Solution: The current through the system is zero amperes due to the open circuit, resulting in a 0 V drop across each resistor.
Both resistors can therefore be replaced by short circuits, as shown in Fig. 33. Voltage V_{ab} is then directly across the 10 V battery, and

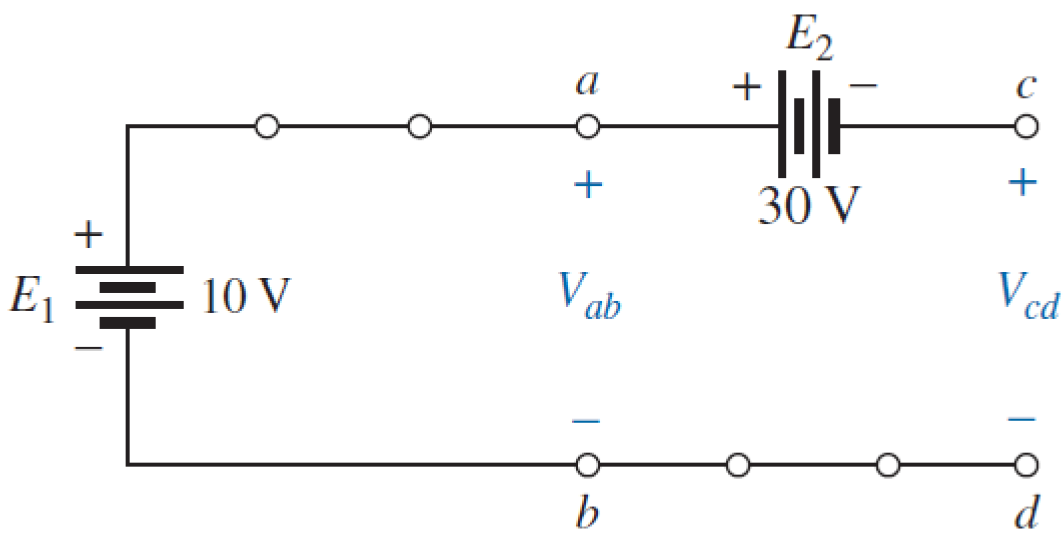


Fig. 33 Circuit in Fig. 32 redrawn.

$$V_{ab} = E_1 = \mathbf{10\ V}$$

Voltage V_{cd} requires an application of Kirchhoff's voltage law:

$$+E_1 - E_2 - V_{cd} = 0$$

$$V_{cd} = E_1 - E_2 = 10\text{ V} - 30\text{ V} = \mathbf{-20\ V}$$

The negative sign in the solution indicates that the actual voltage V_{cd} has the opposite polarity of that appearing in Fig. 32.

EXAMPLE 17 Determine V and I for the network in Fig. 34 if resistor R_2 is shorted out.

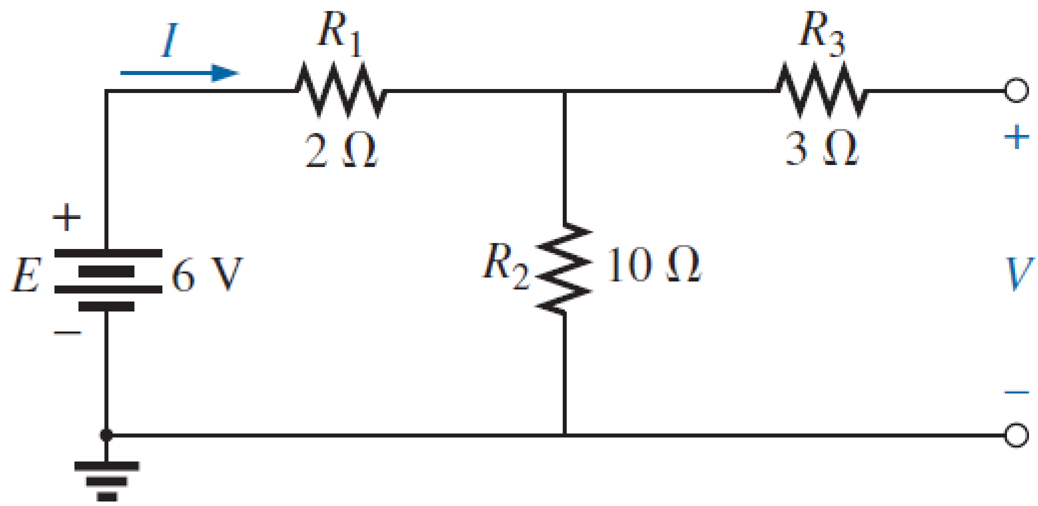


Fig. 34

Solution: The redrawn network appears in Fig. 35.

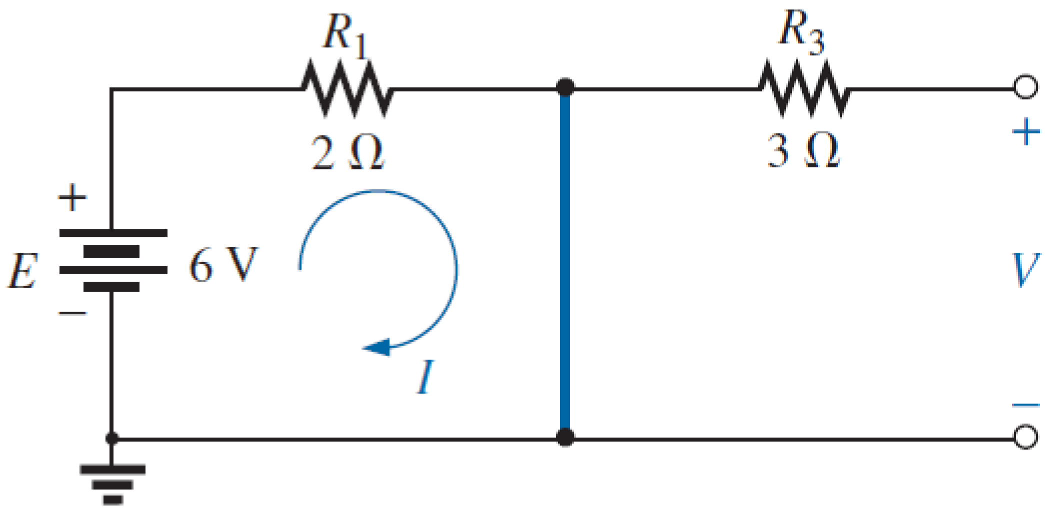


Fig. 35 Network in Fig. 34 with R_2 replaced by a jumper.

The current through the 3 ohm resistor is zero due to the open circuit, causing all the current *I* to pass through the jumper. Since

$$V_{3\Omega} = IR = (0)R = 0\ \text{V}$$

, the voltage *V* is directly across the short, and

$$V = \mathbf{0\ V}$$

$$I = \frac{E}{R_1} = \frac{6\ \text{V}}{2\ \Omega} = \mathbf{3\ A}$$

1.7 SUMMARY TABLE

Now that the series and parallel configurations have been covered in detail, we will review the salient equations and characteristics of each. The equations for the two configurations have a number of similarities. In fact, the equations for one can often be obtained directly from the other by simply applying the **duality** principle.

TABLE 1

Summary table.

Series and Parallel Circuits		
Series	**Duality**	**Parallel**
$R_T = R_1 + R_2 + R_3 + \cdots + R_N$	$R \rightleftarrows G$	$G_T = G_1 + G_2 + G_3 + \cdots + G_N$
R_T increases (G_T decreases) if additional resistors are added in series	$R \rightleftarrows G$	G_T increases (R_T decreases) if additional resistors are added in parallel
Special case: two elements	$R \rightleftarrows G$	$G_T = G_1 + G_2$
$R_T = R_1 + R_2$		and $R_T = \dfrac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2}} = \dfrac{R_1R_2}{R_1 + R_2}$
I the same through series elements	$I \rightleftarrows V$	V the same across parallel elements
$E = V_1 + V_2 + V_3$	$E, V \rightleftarrows I$	$I_T = I_1 + I_2 + I_3$
Largest V across largest R	$V \rightleftarrows I$ and $R \rightleftarrows G$	Greatest I through largest G (smallest R)
$V_x = \dfrac{R_x E}{R_T}$	$E, V \rightleftarrows I$ and $R \rightleftarrows G$	$I_x = \dfrac{G_x I_T}{G_T} = \dfrac{R_T I_T}{R_x}$
		with $I_1 = \dfrac{R_2 I_T}{R_1 + R_2}$ and $I_2 = \dfrac{R_1 I_T}{R_1 + R_2}$
$P = EI_T$	$E \rightleftarrows I$ and $I \rightleftarrows E$	$P = I_T E$
$P = I^2R$	$I \rightleftarrows V$ and $R \rightleftarrows G$	$P = V^2G = V^2/R$
$P = V^2/R$	$V \rightleftarrows I$ and $R \rightleftarrows G$	$P = I^2/G = I^2R$

Please read and try understand in the first reference Chapter 6.

References:

1- Introductory Circuits Analysis, By Boylested, Tenth (10th) Edition.

2- Schaum's Outline of Theory and Problems of Basic Circuit Analysis, By John O'Malley, Second (2nd) Edition.

3- Any reference that has Direct Current Circuits Analysis (DCCA).

Direct Current Circuits Analysis (DCCA)
LE104
2017-2018

Doctor
Sarmad Fawzi

Chapter 5

SERIES-PARALLEL NETWORKS

SERIES-PARALLEL NETWORKS

A firm understanding of the basic principles associated with series and parallel circuits is a sufficient background to begin an investigation of **any single-source dc network** having a combination of series and parallel elements or branches. Multisource networks are considered in detail in Chapters 8 and 9. In general,

Series-Parallel networks are networks that contain both series and parallel circuit configurations.

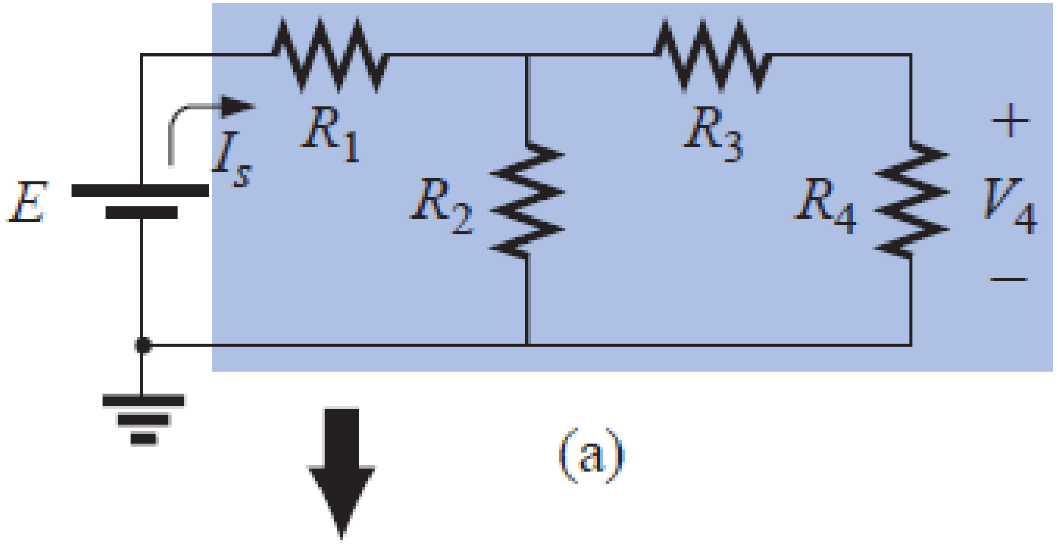
One can become proficient in the analysis of series-parallel networks only through:
1) exposure, 2) practice, and 3) experience. In time the path to the desired unknown becomes more obvious as one recalls similar configurations and the frustration resulting from choosing the wrong approach. There are a few steps that can be helpful in getting started on the first few exercises, although the value of each will become apparent only with experience.

General Approach

1. **Take a moment to study the problem "in total" and make a brief mental sketch of the overall approach you plan to use**. The result may be time- and energy-saving shortcuts.
2. **Next examine each region of the network independently before tying them together in series-parallel combinations.** This will usually simplify the network and possibly reveal a direct approach toward obtaining one or more desired unknowns. It also eliminates many of the errors that might result due to the lack of a systematic approach.
3. **Redraw the network as often as possible** with the reduced branches and undisturbed unknown quantities to maintain clarity and provide the reduced networks for the trip back to unknown quantities from the source.
4. **When you have a solution, check that it is reasonable** by considering the magnitudes of the energy source and the elements in the network. If it does not seem reasonable, either solve the circuit using another approach or check over your work very carefully.

Reduce and Return Approach

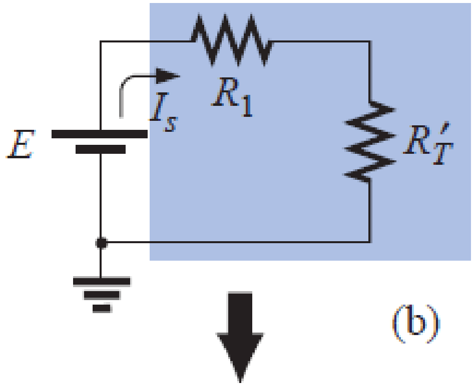
For many single-source, series-parallel networks, the analysis is one that works back to the source, **determines the source current**, and then finds its **way to the desired unknown.**



(a)

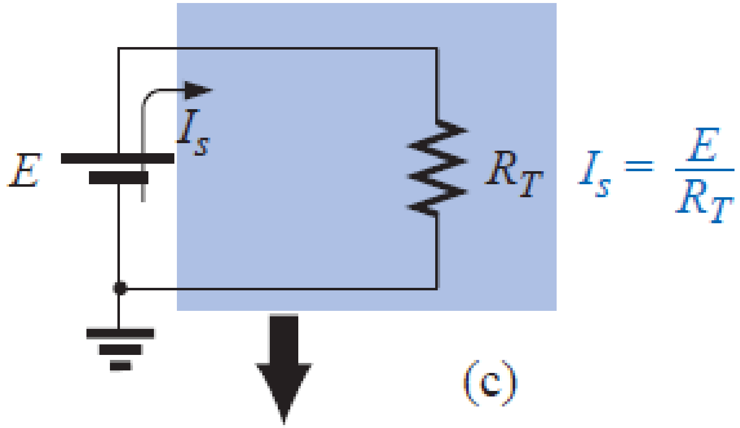
In Fig. 1(a), for instance, the voltage V_4 is desired. The absence of a single series or parallel path to V_4 from the source immediately:

First, series and parallel elements must be combined to establish the reduced circuit of Fig. 1(b).



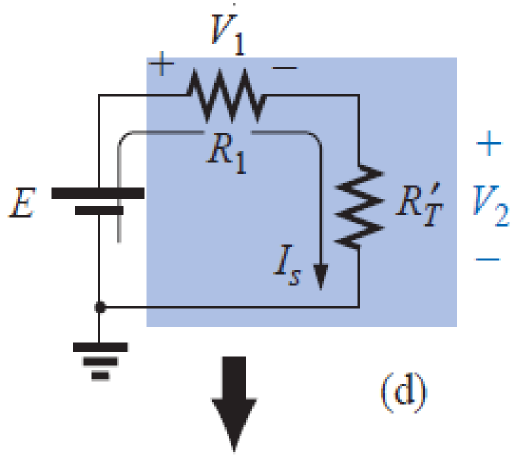
(b)

Second, series elements are combined to form the simplest of configurations in Fig. 1(c). The source current can now be determined using **Ohm's law**,



(c)

Third, and we can proceed back through the network as shown in Fig. 1(d). The voltage V_2 can be determined



(d)

Finally, and then the original network can be redrawn, as shown in Fig. 1(e). Since V_2 is now known, the **voltage divider rule (VDR)** can be used to find the desired voltage V_4.

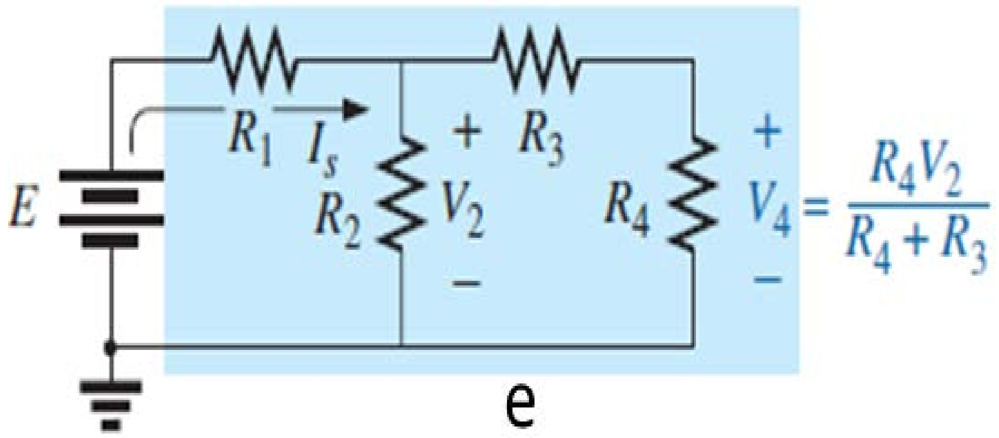


Fig. 1

Introducing the reduce and return approach.

Block Diagram Approach

The block diagram approach will be employed throughout to emphasize the fact that combinations of elements, not simply single resistive elements, can be in series or parallel.

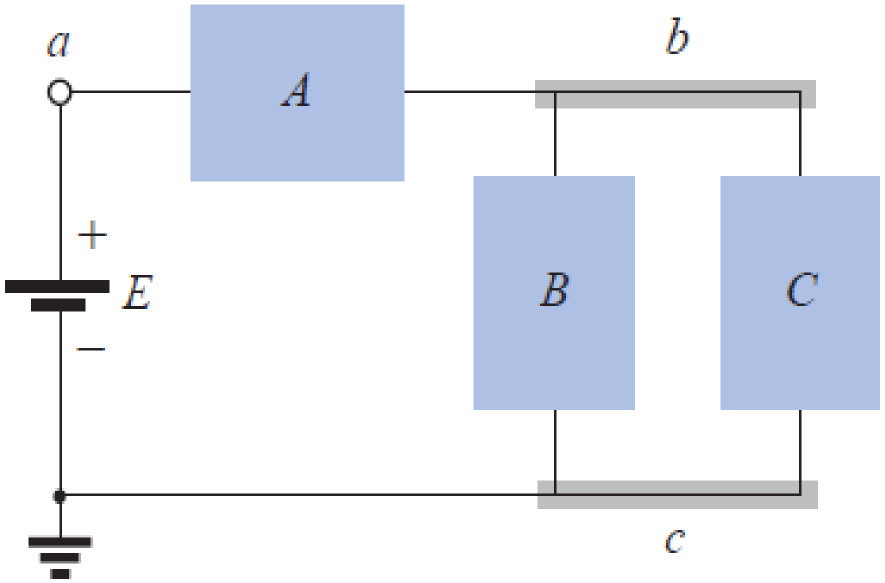


Fig. 2

Introducing the block diagram approach.

1) In Fig. 2, blocks *B* and *C* are in parallel (points *b* and *c* in common),
2) and the voltage source *E* is in series with block *A* (point *a* in common).

3) The parallel combination of *B* and *C* is also in series with *A* and the voltage source *E* due to the common points *b* and *c*, respectively.

To ensure that the analysis to follow is as clear and uncluttered as possible,

For series resistors R_1 and R_2, a **comma** will be inserted between their **subscript notations**, as shown here:

$$\boldsymbol{R_{1,2} = R_1 + R_2}$$

For parallel resistors R_1 and R_2, **the parallel symbol** will be inserted between their **subscript notations**, as follows:

$$R_{1\|2} = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2}$$

Example 1: Calculate the source current (I_S); (I_B) ;(I_C) for the cct. of fig (3):

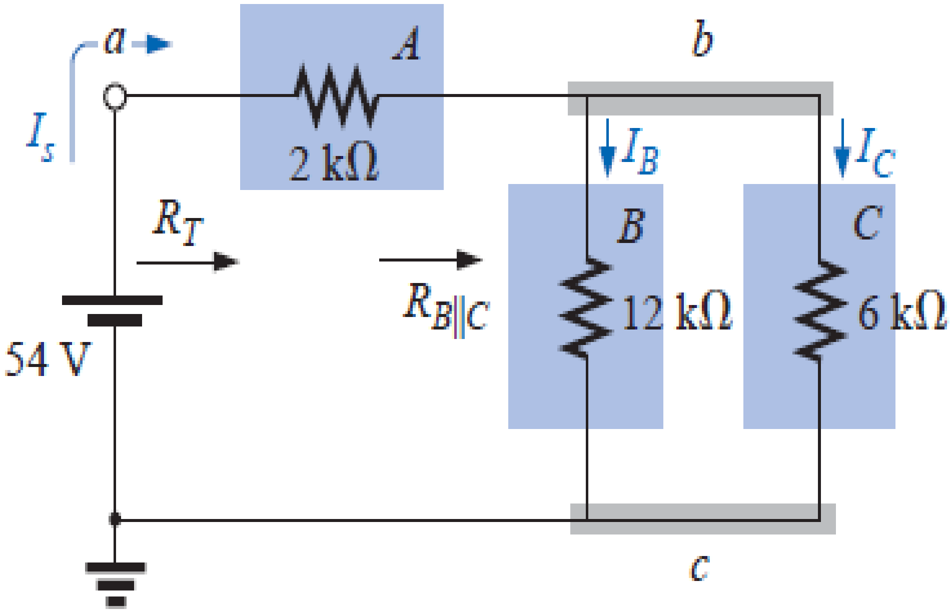


Fig. (3)

Sol: The parallel combination of R_B and R_C results in

$$R_{B\|C} = R_B \,\|\, R_C = \frac{(12\ \text{k}\Omega)(6\ \text{k}\Omega)}{12\ \text{k}\Omega + 6\ \text{k}\Omega} = 4\ \text{k}\Omega$$

The equivalent resistance $R_{B\|C}$ is then in series with R_A, and the total resistance "seen" by the source is

$$\begin{aligned} R_T &= R_A + R_{B\|C} \\ &= 2\ \text{k}\Omega + 4\ \text{k}\Omega = 6\ \text{k}\Omega \end{aligned}$$

The result is an equivalent network, as shown in Fig. 4, permitting the determination of the source current I_s.

$$I_s = \frac{E}{R_T} = \frac{54\ \text{V}}{6\ \text{k}\Omega} = \mathbf{9\ mA}$$

and, since the source and R_A are in series,

$$I_A = I_s = 9\ \text{mA}$$

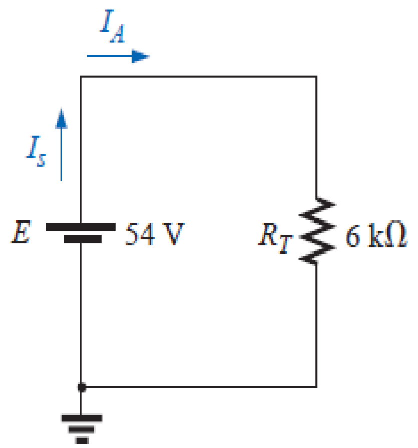


Fig. (4)

We can then use the equivalent network of Fig. (5) to determine I_B and I_C using the **current divider rule (CDR):**

$$I_B = \frac{6\ \text{k}\Omega(I_s)}{6\ \text{k}\Omega + 12\ \text{k}\Omega} = \frac{6}{18}I_s = \frac{1}{3}(9\ \text{mA}) = \mathbf{3\ mA}$$

$$I_C = \frac{12\ \text{k}\Omega(I_s)}{12\ \text{k}\Omega + 6\ \text{k}\Omega} = \frac{12}{18}I_s = \frac{2}{3}(9\ \text{mA}) = \mathbf{6\ mA}$$

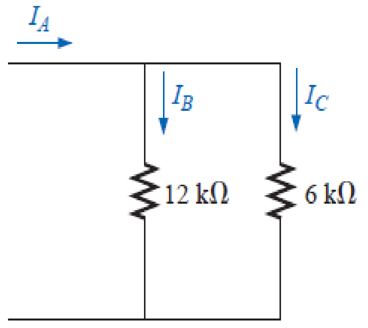


Fig. (5)

or, applying Kirchhoff's current law,

$$I_C = I_s - I_B = 9\ \text{mA} - 3\ \text{mA} = \mathbf{6\ mA}$$

Note that in this solution, we worked back to the source to obtain the source current or total current supplied by the source.

Example 2: for the cct. of fig. (6) calculate: R_A , R_B , R_C , R_T , I_T , I_A , I_B , I_C , I_{R1} , I_{R2} , I_{R3} , I_{R4} , I_{R5}

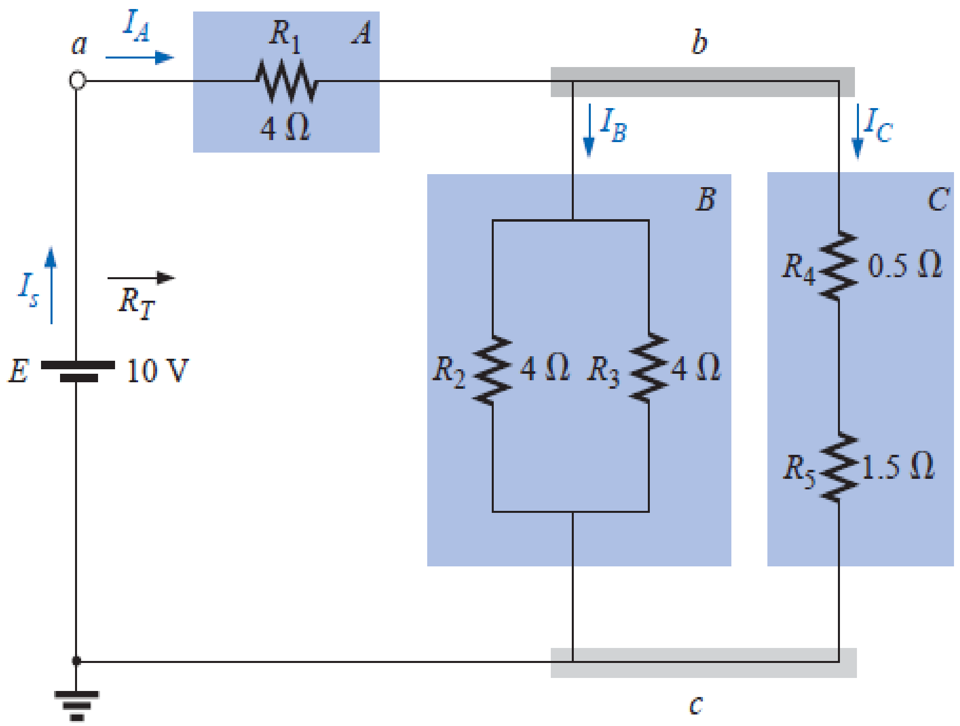


Fig.(6)

Sol. : By re-drawing the cct. of fig.(6) we get an equivalent cct. of fig(7):

$$A: \quad R_A = 4\ \Omega$$

$$B: \quad R_B = R_2 \parallel R_3 = R_{2\parallel 3} = \frac{R}{N} = \frac{4\ \Omega}{2} = 2\ \Omega$$

$$C: \quad R_C = R_4 + R_5 = R_{4,5} = 0.5\ \Omega + 1.5\ \Omega = 2\ \Omega$$

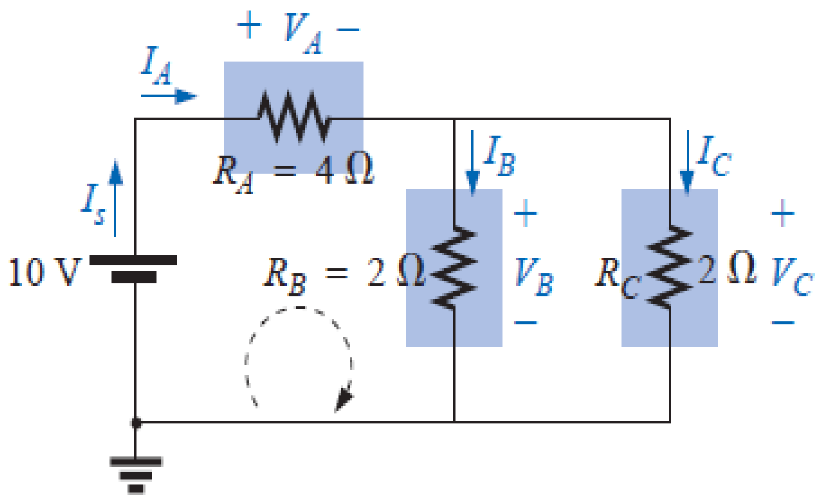


Fig. (7)

Blocks B and C are still in parallel, and

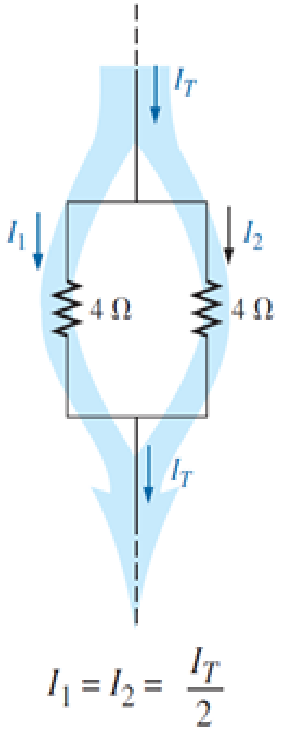
$$R_{B\|C} = \frac{R}{N} = \frac{2\ \Omega}{2} = 1\ \Omega$$

$$\begin{aligned} R_T &= R_A + R_{B\|C} \\ &= 4\ \Omega + 1\ \Omega = \mathbf{5\ \Omega} \end{aligned}$$

$$I_s = \frac{E}{R_T} = \frac{10\ \text{V}}{5\ \Omega} = \mathbf{2\ A}$$

$$I_A = I_s = \mathbf{2\ A}$$

$$I_B = I_C = \frac{I_A}{2} = \frac{I_s}{2} = \frac{2\ \text{A}}{2} = \mathbf{1\ A}$$



Returning to the network of Fig. 6, we have:

$$I_{R_2} = I_{R_3} = \frac{I_B}{2} = \mathbf{0.5\ A}$$

The voltages V_A, V_B, and V_C from either figure are

$$V_A = I_A R_A = (2\ \text{A})(4\ \Omega) = \mathbf{8\ V}$$
$$V_B = I_B R_B = (1\ \text{A})(2\ \Omega) = \mathbf{2\ V}$$
$$V_C = V_B = \mathbf{2\ V}$$

Applying Kirchhoff's voltage law for the loop indicated in Fig. (7), we obtain:

$$\Sigma_{\circlearrowright} V = E - V_A - V_B = 0$$
$$E = V_A + V_B = 8\ \text{V} + 2\ \text{V}$$
$$\underline{10\ \text{V} = 10\ \text{V} \quad \text{(checks)}}$$

Example 3 : for the circuit in the fig(8) find: R_T , R_A , R_B , R_C , I_S , I_A , I_B , I_C , I_1 , I_2

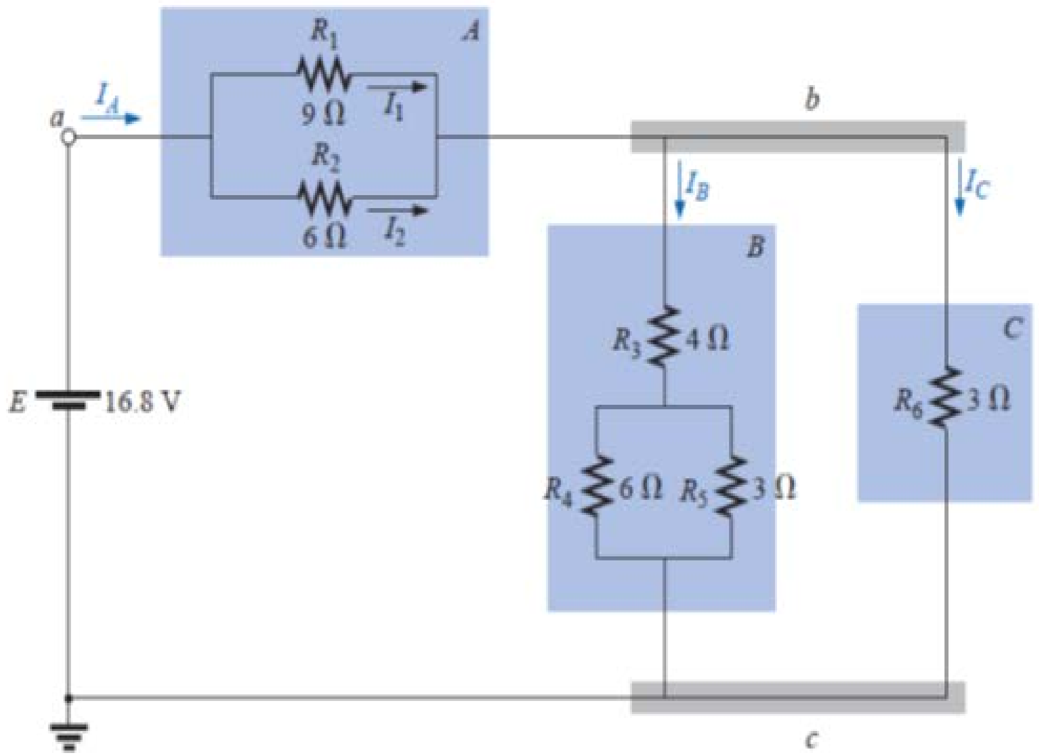


Fig. (8)

SOL.

$$R_A = R_{1\|2} = \frac{(9\ \Omega)(6\ \Omega)}{9\ \Omega + 6\ \Omega} = \frac{54\ \Omega}{15} = 3.6\ \Omega$$

$$R_B = R_3 + R_{4\|5} = 4\ \Omega + \frac{(6\ \Omega)(3\ \Omega)}{6\ \Omega + 3\ \Omega} = 4\ \Omega + 2\ \Omega = 6\ \Omega$$

$$R_C = 3\ \Omega$$

The network of Fig. (8) can then be redrawn in reduced form, as shown in Fig. (9).

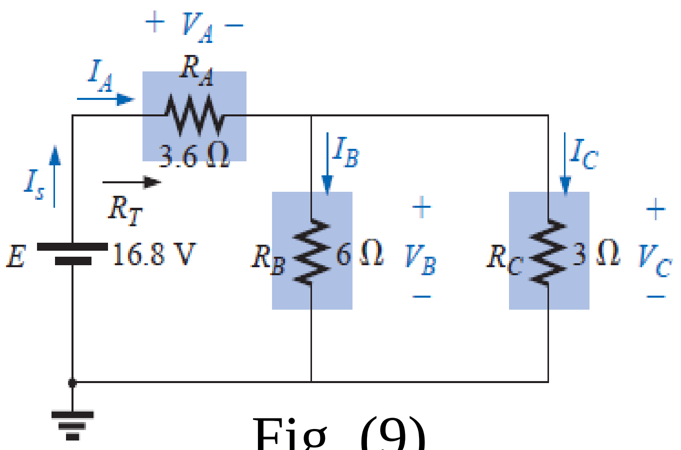


Fig. (9)

$$R_T = R_A + R_{B\|C} = 3.6\ \Omega + \frac{(6\ \Omega)(3\ \Omega)}{6\ \Omega + 3\ \Omega}$$

$$= 3.6\ \Omega + 2\ \Omega = \mathbf{5.6\ \Omega}$$

$$I_s = \frac{E}{R_T} = \frac{16.8\ \text{V}}{5.6\ \Omega} = \mathbf{3\ A}$$

$$I_A = I_s = \mathbf{3\ A}$$

Applying the current divider rule yields

$$I_B = \frac{R_C I_A}{R_C + R_B} = \frac{(3\ \Omega)(3\ \text{A})}{3\ \Omega + 6\ \Omega} = \frac{9\ \text{A}}{9} = \mathbf{1\ A}$$

By Kirchhoff's current law,

$$I_C = I_A - I_B = 3\ \text{A} - 1\ \text{A} = \mathbf{2\ A}$$

By Ohm's law,

$$V_A = I_A R_A = (3\ \text{A})(3.6\ \Omega) = \mathbf{10.8\ V}$$

$$V_B = I_B R_B = V_C = I_C R_C = (2\ \text{A})(3\ \Omega) = \mathbf{6\ V}$$

Returning to the original network (Fig. 8) and applying the current divider rule:

$$I_1 = \frac{R_2 I_A}{R_2 + R_1} = \frac{(6\ \Omega)(3\ \text{A})}{6\ \Omega + 9\ \Omega} = \frac{18\ \text{A}}{15} = \mathbf{1.2\ A}$$

By Kirchhoff's current law,

$$I_2 = I_A - I_1 = 3\ \text{A} - 1.2\ \text{A} = \mathbf{1.8\ A}$$

EXAMPLE 4: Find the indicated currents and voltages for the network of Fig.(8)

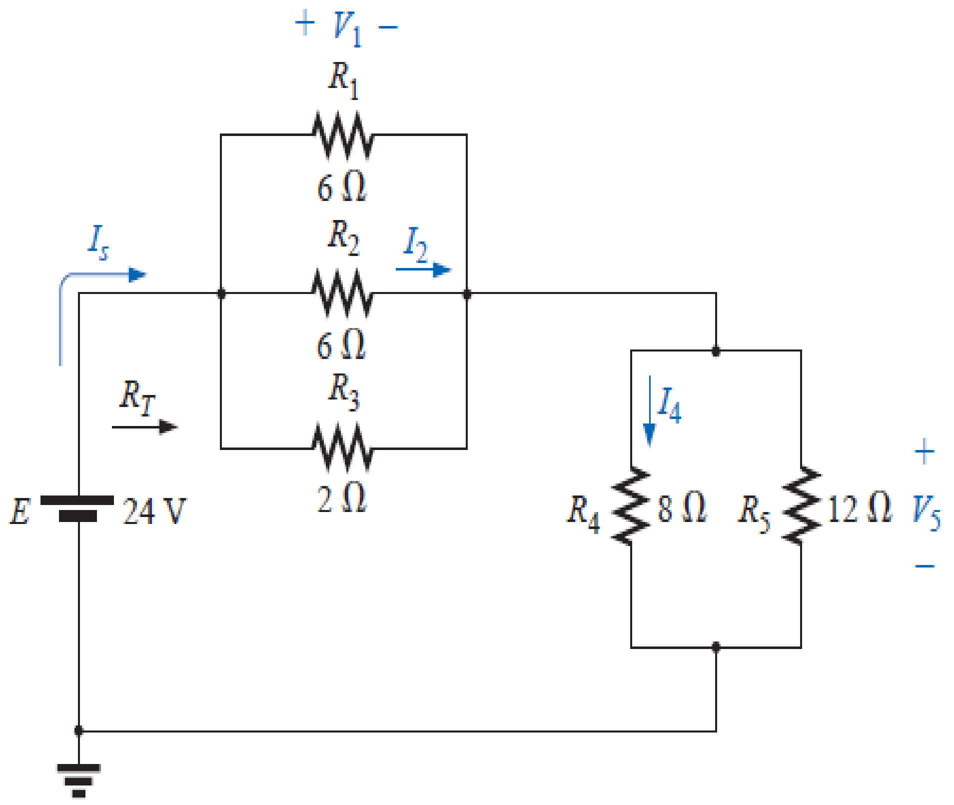
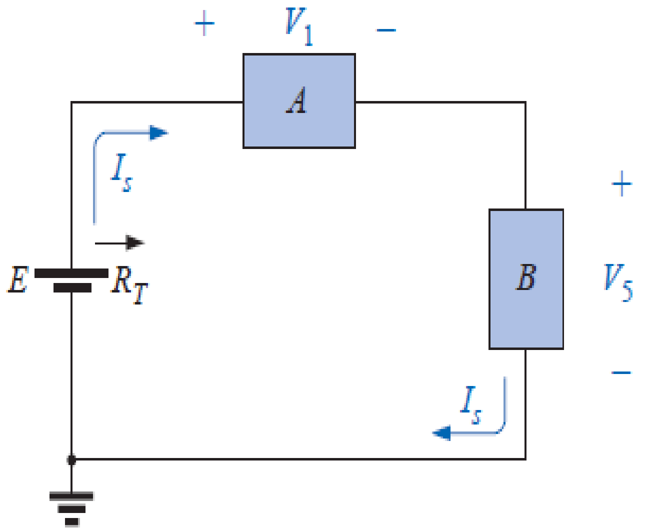


Fig.(8)



Block diagram for Figure (9)

$$R_{1\|2} = \frac{R}{N} = \frac{6\ \Omega}{2} = 3\ \Omega$$

$$R_A = R_{1\|2\|3} = \frac{(3\ \Omega)(2\ \Omega)}{3\ \Omega + 2\ \Omega} = \frac{6\ \Omega}{5} = 1.2\ \Omega$$

$$R_B = R_{4\|5} = \frac{(8\ \Omega)(12\ \Omega)}{8\ \Omega + 12\ \Omega} = \frac{96\ \Omega}{20} = 4.8\ \Omega$$

The reduced form of Fig. (8)will then appear as shown in Fig.(10) :

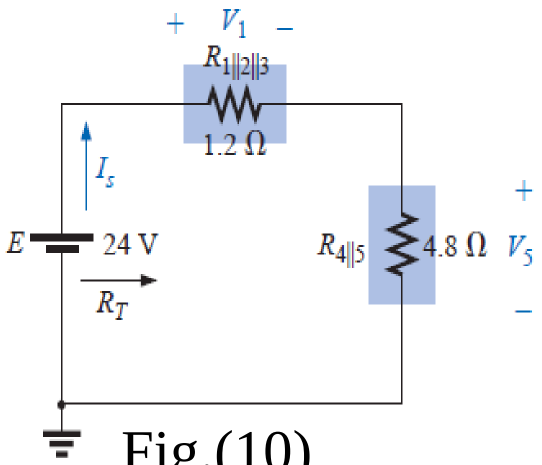


Fig.(10)

$$R_T = R_{1\|2\|3} + R_{4\|5} = 1.2\ \Omega + 4.8\ \Omega = \mathbf{6\ \Omega}$$

$$I_s = \frac{E}{R_T} = \frac{24\ \text{V}}{6\ \Omega} = \mathbf{4\ A}$$

with
$$V_1 = I_s R_{1\|2\|3} = (4\ \text{A})(1.2\ \Omega) = \mathbf{4.8\ V}$$
$$V_5 = I_s R_{4\|5} = (4\ \text{A})(4.8\ \Omega) = \mathbf{19.2\ V}$$

Applying Ohm's law,

$$I_4 = \frac{V_5}{R_4} = \frac{19.2\ \text{V}}{8\ \Omega} = \mathbf{2.4\ A}$$

$$I_2 = \frac{V_2}{R_2} = \frac{V_1}{R_2} = \frac{4.8\ \text{V}}{6\ \Omega} = \mathbf{0.8\ A}$$

Example 5 :

a. Find the voltages V_1, V_3, and V_{ab} for the network of Fig. (11).

b. Calculate the source current I_S.

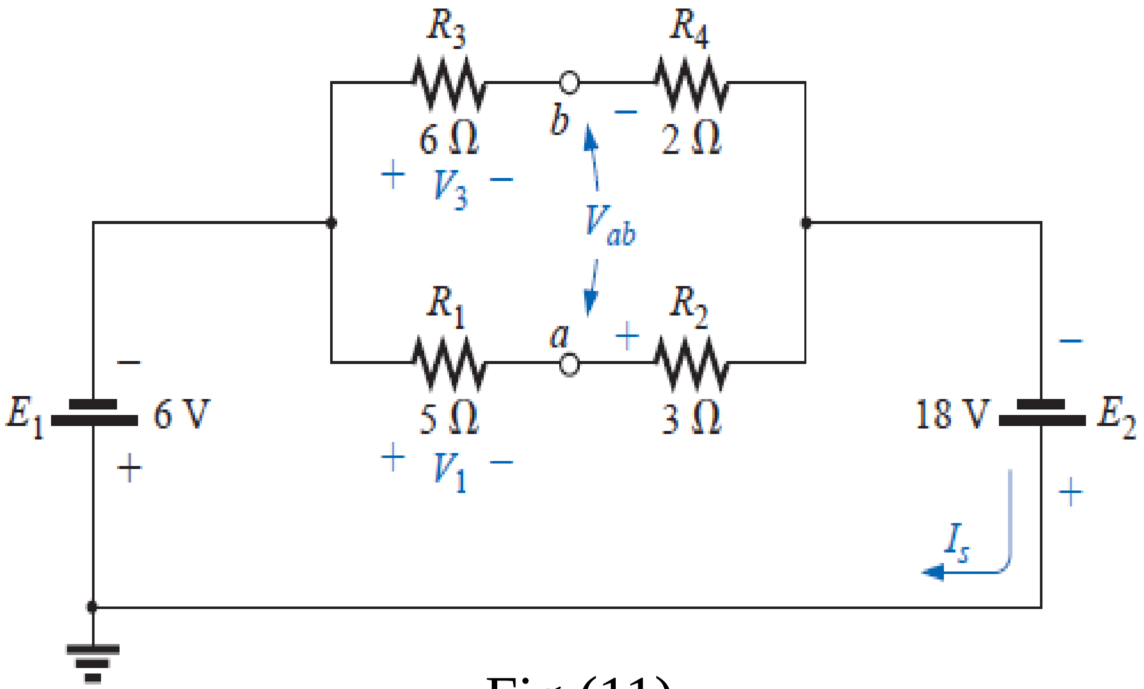


Fig.(11)

Sol: Since combining both sources will not affect the unknowns

The cct. is re-drawn as in fig(12):

The net source voltage is the difference between the two with the polarity of the larger.

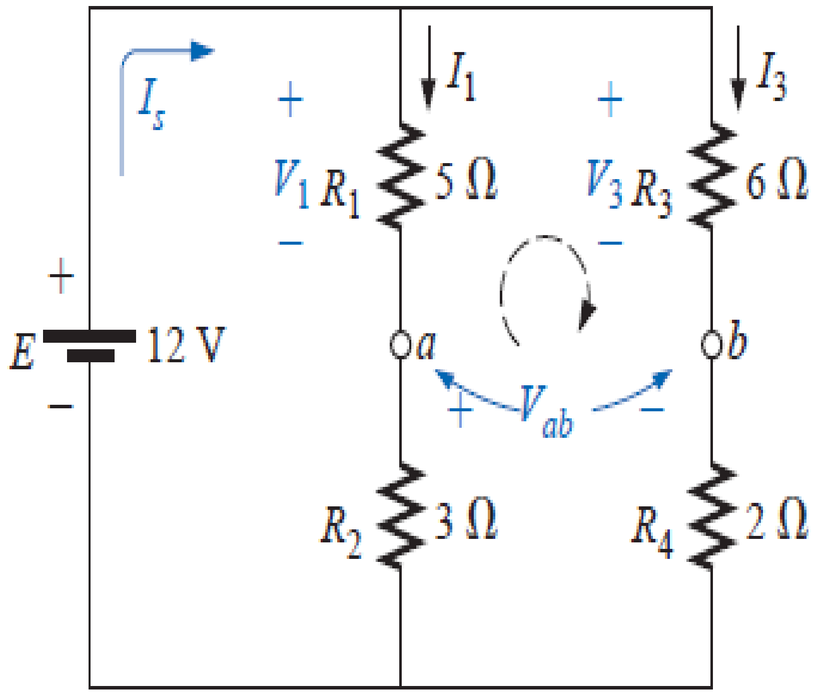


Fig.(12)

a.

$$V_1 = \frac{R_1 E}{R_1 + R_2} = \frac{(5\ \Omega)(12\ \text{V})}{5\ \Omega + 3\ \Omega} = \frac{60\ \text{V}}{8} = \mathbf{7.5\ V}$$

$$V_3 = \frac{R_3 E}{R_3 + R_4} = \frac{(6\ \Omega)(12\ \text{V})}{6\ \Omega + 2\ \Omega} = \frac{72\ \text{V}}{8} = \mathbf{9\ V}$$

The open-circuit voltage V_{ab} is determined by applying Kirchhoff's voltage law around the indicated loop of Fig.(12) in the clockwise direction starting at terminal *a*.

$$+V_1 - V_3 + V_{ab} = 0$$

and $$V_{ab} = V_3 - V_1 = 9\text{ V} - 7.5\text{ V} = \mathbf{1.5\ V}$$

b. By Ohm's law,

$$I_1 = \frac{V_1}{R_1} = \frac{7.5\text{ V}}{5\ \Omega} = 1.5\text{ A}$$

$$I_3 = \frac{V_3}{R_3} = \frac{9\text{ V}}{6\ \Omega} = 1.5\text{ A}$$

Applying Kirchhoff's current law,

$$I_s = I_1 + I_3 = 1.5\text{ A} + 1.5\text{ A} = \mathbf{3\ A}$$

Please read and try understand in the first reference Chapter 7.

References:

1- Introductory Circuits Analysis, By Boylested, Tenth (10^{th}) Edition.

2- Schaum's Outline of Theory and Problems of Basic Circuit Analysis, By John O'Malley, Second (2nd) Edition.

3- Any reference that has a Direct Current Circuits Analysis (DCCA).

Thank you for listening

Math. Functions

pow (real) <math.h>

Power function, x to the y
Declaration:
double pow(double x, double y);

Q) Write a program to calculate the cubic number for 4 different numbers

```
#include <iostream.h>
#include <conio.h>
#include <math.h>
int main()
{
double n,c;
clrscr();

  for (int i=1;i<=4;i=i+1)
    {
     cout <<"Number:=";
     cin>> n;
     c=pow(n,3);
    cout <<"Cubic Number:="<<c;
    }
cout << "\n\nHit any key to continue";
getch();
return 0;
}
```

cos, sin, tan (real) <math.h>

Cosine, sine, and tangent functions

Declaration:
double cos(double x);
double sin(double x);
double tan(double x);

Remarks:
- cos compute the cosine of the input value
- sin compute the sine of the input value
- tan calculate the tangent of the input value

Angles are specified in radians.

```
#include <stdio.h>
#include <math.h>

int main(void)
{
   double result, x;

   x = 90;
   result = sin(x*3.14/180);
   cout <<"The sin of “<<x<<” is "<< result;
//the oputput is: The sin of 90.00 is 1.00
   return 0;
}
```

acos, asin, atan, atan2 (real) <math.h>

Arc cosine, arc sine, and arc tangent functions

Declaration:

double acos(double x);
double asin(double x);
double atan(double x);
double atan2(double y, double x);

Remarks:

- acos of a real value compute the arc cosine of that value
- asin of a real value compute the arc sine of that value
- atan calculate the arc tangent of the input value
- atan2 also calculate the arc tangent of the input value

Real arguments to acos, and asin must be in the range -1 to 1.

```
#include <stdio.h>
#include <math.h>

int main(void)
{
   double result;
   double x = 1.00;

   result = asin(x)/(3.14/180);
  cout <<"The arc sin of “<<x<<” is "<< result;
//the oputput is: The arc sin of 1.00 is 90.00

   return(0);
}
```

exp (real) <math.h>

- Real exp calculates e to the xth power

Declaration:

double exp(double x);

Remarks:

exp calculates the exponential function e**x.

```
#include <stdio.h>
#include <math.h>

int main(void)
{
   double result;
   double x = 4.0;

   result = exp(x);
cout <<" 'e' raised to the power of "<<x<<" (e ^ "<<x<<") = "<< result;
      return 0;
}
The output is: 'e' raised to the power of 4.000000e^4.000000)=54.598150
```

sqrt (real) < math.h >

Calculates square root

Declaration:
- Real: double sqrt(double x);
  long double sqrtl(long double @E(x));

Remarks:
sqrt calculates the positive square root of the input value.

```
#include <math.h>
 #include <iostream.h>

 int main(void)
 {
   double x = 4.0, result;

   result = sqrt(x);
   cout <<"The square root of  "<<x<<" is "<< result;
   //the oputput is: The square root of 4.000000 is 2.000000
        return 0;
}
```

abs <math.h, stdlib.h, complex.h>
fabs < math.h >
labs < math.h, stdlib.h >

- abs (a macro) gets the absolute value of an integer
- fabs calculate the absolute value of a floating-point number
- labs calculates the absolute value of a long number

Declaration:
- abs
  int abs(int x);
- double fabs(double x);
- long int labs(long int x);

```
#include <iostream.h>
#include <math.h>

int main(void)
{
  int number = -1234;

  cout <<"number: "<<number<<" absolute value: " << abs(number);
  //the oputput is: number: -1234 absolute value: 1234
  return 0;
}
```

randomize **<stdlib.h>**

Initializes random number generator. time.h must included

random **< stdlib.h >**

Returns a random number: random(num);
random returns a random number between 0 and (num-1).

```
#include <stdlib.h>
#include <iostream.h>
#include <time.h>
/* prints a random number in the range 0 to 99 */
int main(void)
{
   randomize();
   cout <<"Random number in the 0-99 range: "<< random (100);
   return 0;
}
```

Q) Write a program to solve the following equations, where Q in degree

$$y = \begin{cases} |\tan Q - e^{Q}| & x < 0 \\ \dfrac{-b + \sqrt{b^2 - 4ac}}{2a} & x \geq 0 \end{cases}$$

```
#include <iostream.h>
#include <conio.h>
#include <math.h>
int main()
{
 double y,x,a,b,c,Q;
 clrscr();
 cout <<"x="; cin >>x;
 if (x<0)
 {
   cout <<"Q="; cin >>Q;
   y=fabs(tan(Q*3.14/180)-exp(Q));
 }
 else
 {
  cout <<"a="; cin >>a;
  cout <<"b="; cin >>b;
  cout <<"c="; cin >>c;
  double bs=b*b-4*a*c;
  if ((a) && (bs>0))
   y=(-b+sqrt(bs))/2*a;
  else
   {
    cout <<"Error illegal values";
    getch();
    return 1;
   }
 }
 cout <<"y="<<y;
 getch();
 return 0;
}
```