Introductory Lecture

First, What are **standing waves.**

•What are the conditions for creating standing waves.

•How standing waves in a laser cavity are determined by the laser design.

Second, the properties of the optical signal which is amplified while passing back and force through the active medium are discussed.

Third, **longitudinal modes a**re created in the laser cavity. Their importance and methods for controlling them.

Forth, the distribution of energy along the cross section of the beam, which determine the **transverse modes.**

At the end describing the **common optical cavities** and the way to test their stability.

Two waves of the same frequency and amplitude are moving in opposite directions, which is the **condition for creating a standing wave**.

Remember that the electromagnetic waves inside the laser cavity are 3 dimensional, and are moving along the optical axis of the laser.

Create A Standing Wave

- The optical path from one mirror to the other and back must an **integer multiplication of the wavelength**.
- The wave must start with the same phase at the mirror
- The Length between the mirrors is constant (L), the suitable wavelengths, which create standing waves, must fulfill the condition: $l_m = 2L/m$

L = Length of the optical cavity.

m = Number of the mode, which is equal to the number wavelengths inside the optical cavity

l_m = Wavelength of mode m inside the laser cavity.

Wavelength in matter (l_m) is equal to: $l_m = l_0/n$

l_0 = Wavelength of light in vacuum.

n = **Index of refraction** of the active medium.

c = Velocity of light in vacuum.

Wavelength in matter **(l_m)** is equal to: $l_m = l_0/\mathbf{n}$

Since: $\mathbf{c} = l_0 n = \mathbf{n} l_m n_m$

The **frequency** of the longitudinal mode $$\nu_m = \frac{c}{n\lambda_m}$$

Inserting l_m into the last equation: $$\nu_m = m \cdot \left(\frac{c}{2 \cdot n \cdot L} \right)$$

The **first mode of oscillation** : $$\nu_1 = \frac{c}{2 \cdot n \cdot L}$$

This mode is called **basic longitudinal mode**, and it has the **basic frequency of the optical cavity**.

Basic Longitudinal

frequency of longitudinal modes is:

$$\nu_m = m \cdot \left(\frac{c}{2 \cdot n \cdot L} \right)$$

The mathematical expression in parenthesis is the **first mode of oscillation** available for this

$$\nu_1 = \frac{c}{2 \cdot n \cdot L}$$

This mode is called **basic longitudinal mode**, and it have the basic frequency of the optical cavity.

Conclusion:

The frequency of each laser mode is equal to integer (mode number m) times the frequency of the basic longitudinal mode.

From this conclusion it is immediately seen that

The difference between frequencies of adjacent modes (mode spacing) is equal to the basic frequency of the cavity: $(\Delta n) = c/(2nL)$

Attention !

Until now it was assumed that the index of refraction (n) is constant along the optical cavity.

This assumption means that the length of the active medium is equal to the length of the optical cavity.

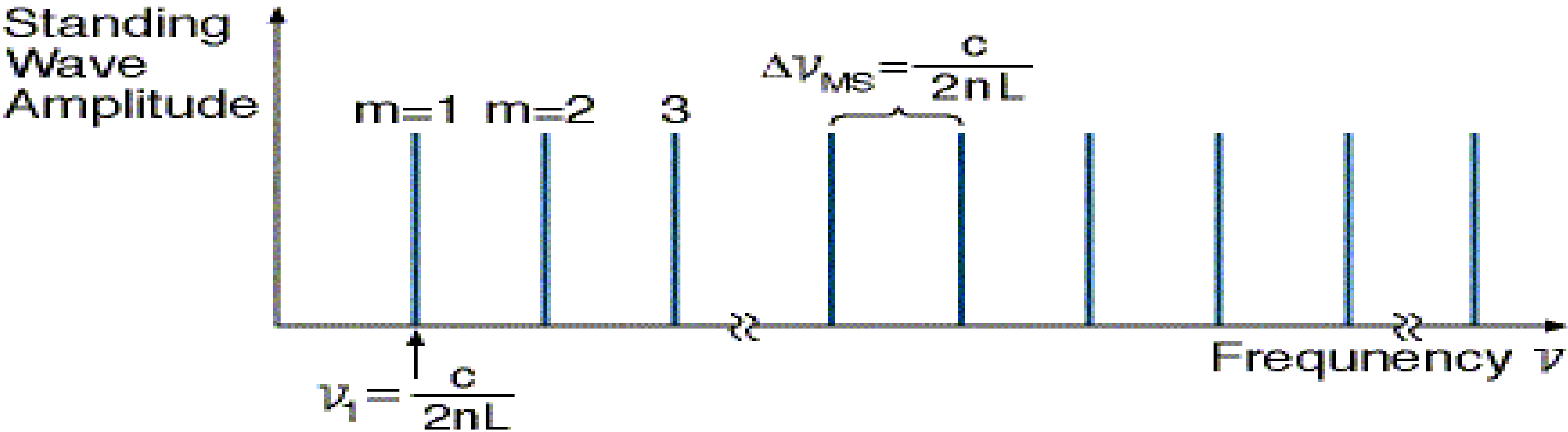
There are lasers in which the mirrors are not at the ends the active medium, so L_1 is not equal to the length of the cavity (L).

In such case each section of the cavity is calculated separately, with its own index of refraction:

MS = Mode Spacing.

$$\Delta\nu_{MS} = \frac{c}{2 \cdot n_1 \cdot L_1 + 2 \cdot n_2 \cdot L_2}$$

Allowed Frequencies inside a Laser Cavity



Allowed Longitudinal modes inside a Laser Cavity of length (L) and index of refraction (n).

In practice, the frequencies are not defined mathematically as single frequencies, but each have a **width of frequencies** around the possible modes,

Longitudinal modes are standing waves along the optical axis of the laser.

The **standing waves** inside a laser are created when the electromagnetic radiation is forced to move back into the cavity from the mirrors.

The allowed frequencies inside an optical cavity are determined by the length of the cavity (L) and the index of refraction of the active medium.

Only those frequencies which create **nodes at both mirrors** are allowed. Thus, the cavity length must be an integer multiplication of half their wavelengths.

The allowed frequencies are spaced at constant interval, which is equal to the basic frequency of the cavity.

The **transverse distribution of intensity**

Are modes in cross section of the beam, perpendicular to the optical axis of the laser.

These **transverse modes** are created by the width of the cavity, which enables a few diagonal modes to develop inside the laser cavity.

A little misalignment of the laser mirrors causes different path length for different rays inside the cavity.

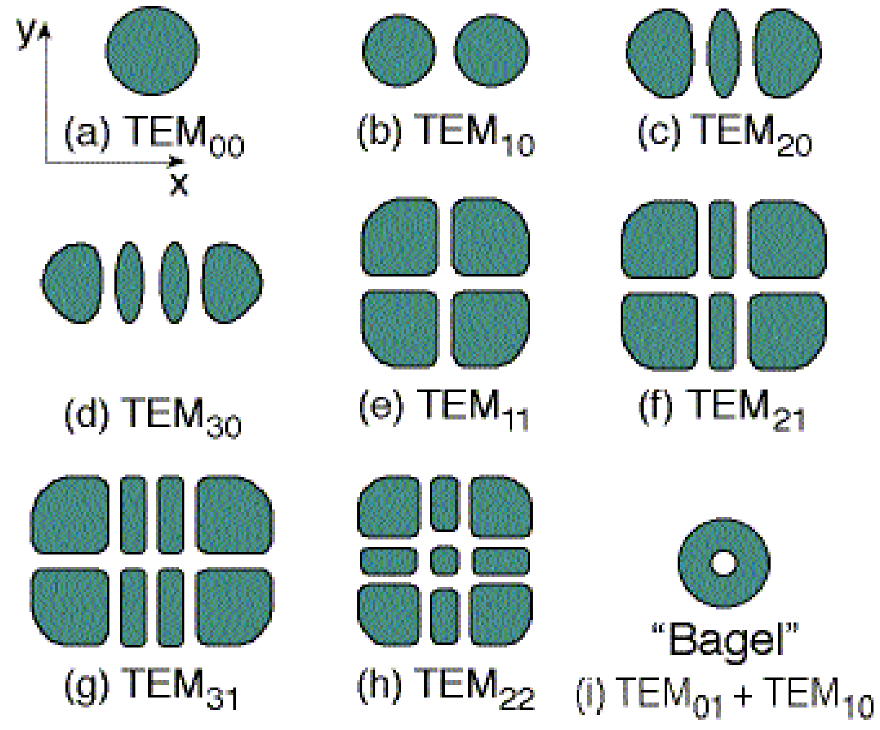
Shape of Transverse Electromagnetic Modes

Transverse Electro-Magnetic (TEM) Modes

The dark areas mark places where laser radiation hit.

When the laser output power is of the order of several

distribution of energy in the beam cross section can be measured by a short illumination of a stick of wood with the laser.



Optical Cavity In every laser cavity

There are different shapes of mirrors, with different lengths between them.
A specific optical cavity is determined by the active medium used, the optical power in it, and the specific application.
The explanation here will summarize the **design principles of an optical cavity:**

Losses inside optical cavity.
Common optical cavities.
Stability criterion of laser optical cavity .

Optical Cavity - Laser Cavity - The region between the end mirrors of the laser.

Optical Axis -The imaginary line connecting the centers of the end mirrors, and perpendicular to them. **The optical axis is in the middle of the optical cavity**.

Aperture -The beam diameter limiting factor inside the laser cavity. Usually the aperture is determined by the diameter of the active medium, but in some lasers a pinhole is inserted into the laser cavity to limit the diameter of the beam. An example is the limiting aperture for achieving single mode operation of the laser

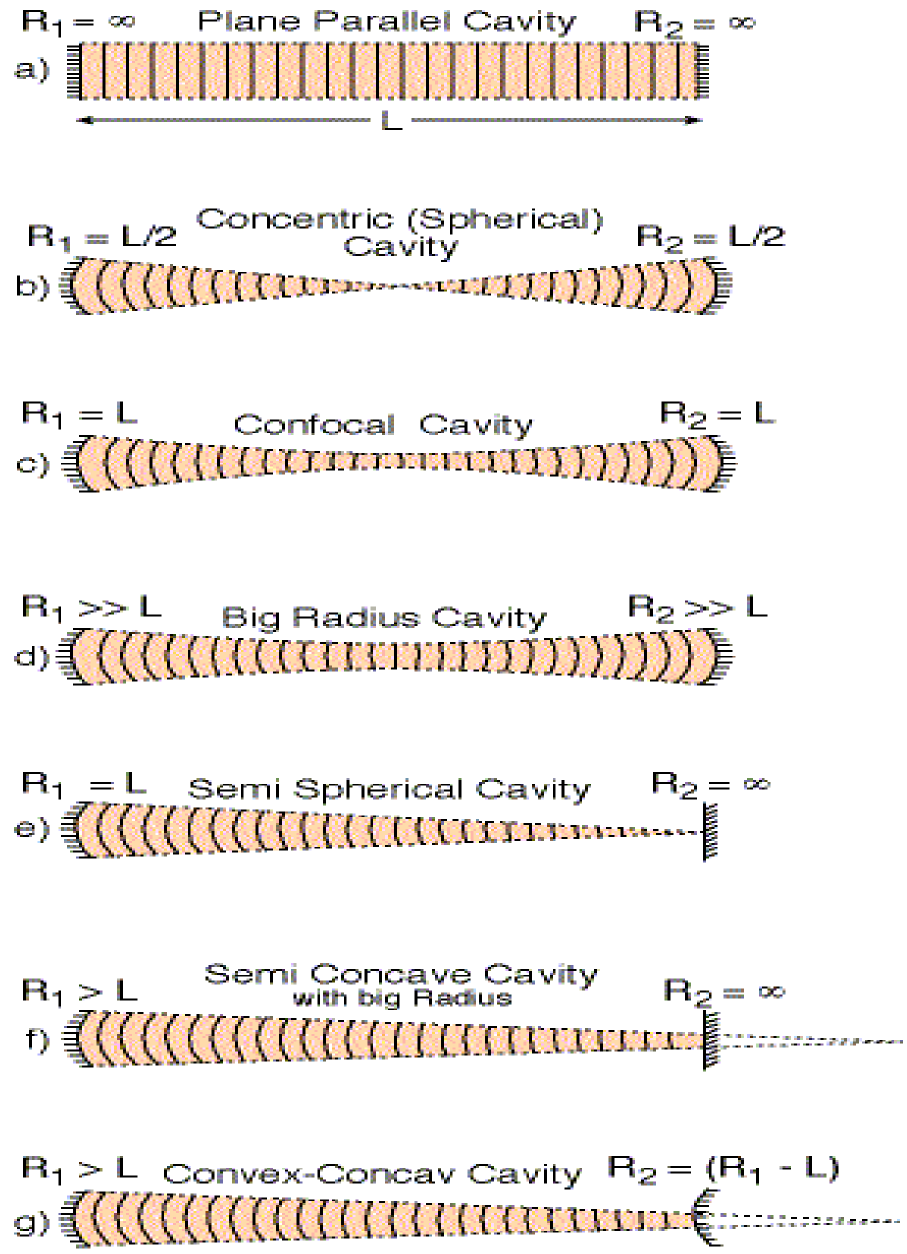
Losses inside Optical Cavity - Include **all the radiation missing from the output of the laser** (emitted through the output coupler).

The gain of the active medium must overcome these losses

Losses inside an optical cavity Misalignment of the laser mirrors –

- **The cavity mirrors are not exactly aligned perpendicular** to the laser axis, and parallel to each other (symmetric), the radiation inside the cavity will not be confined during its path between the mirrors.
- **Absorption, scattering and losses in optical elements** - Since optical elements are not ideal, each interaction with optical element inside the cavity cause some losses.
- **Diffraction Losses** - Every time a laser beam pass through a limiting aperture it diffract. It is not always possible to increase the aperture for reducing the diffraction. As an example, such increase will allow lasing in higher transverse modes which are not desired

The most common optical cavities



Stability Criterion of the cavity A stable cavity

Is a cavity in which the radiation is captured inside the cavity, creating standing waves while the beam move between the mirrors.

The geometry of the cavity determines if the cavity is stable or not.

It is possible to use **unstable resonator** only if the active medium have high gain, since the beam pass through the active medium less times than in stable cavity.

For determining stability of a cavity, a stability criterion need to be defined.

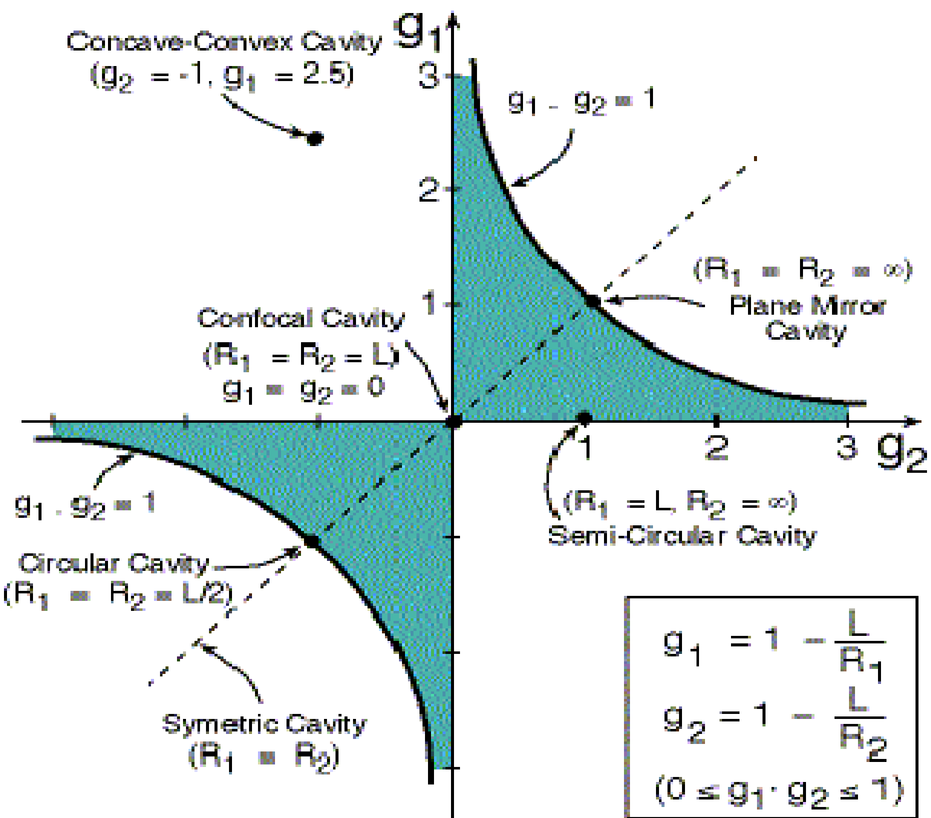
Stability Diagram of an Optical Cavity

The stability criterion for laser cavity is: $\mathbf{0 < g_1 * g_2 < 1}$

$\mathbf{g_1 = 1\text{-}L/R_1}$ $\mathbf{g_2 = 1\text{-}L/R_2}$

In the stability diagram the geometric parameters of the mirrors are the axes x and y.

Figure show the **stability diagram** of all laser cavities.



Laser Gain

The output power of the laser at specific moment is determined by two conflicting factors:

1. **Active medium gain** - which depends on:

a) **Population Inversion** .

b) **Fluorescence line-shape** of the spontaneous emission that is related to the lasing transition .

2. **Losses in the laser**, which include:

a) **Reflections from end mirrors**.

b) **Radiation losses inside the active medium** - due to absorption and scattering.

c) **Diffraction losses** - Due to the finite size of the laser components.

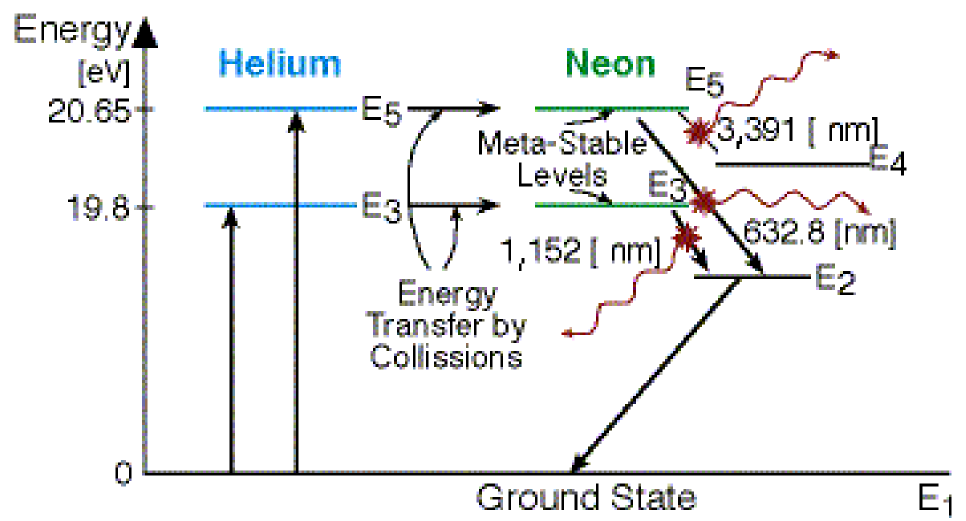
It is clear that a **required condition for lasing** is:

In a round trip path of the radiation between the laser mirrors, the gain must exceed (or at least be equal to) the losses.

Fluorescence line shape of the laser

Laser action inside matter is possible only for those wavelengths for which this material have fluorescent emission.

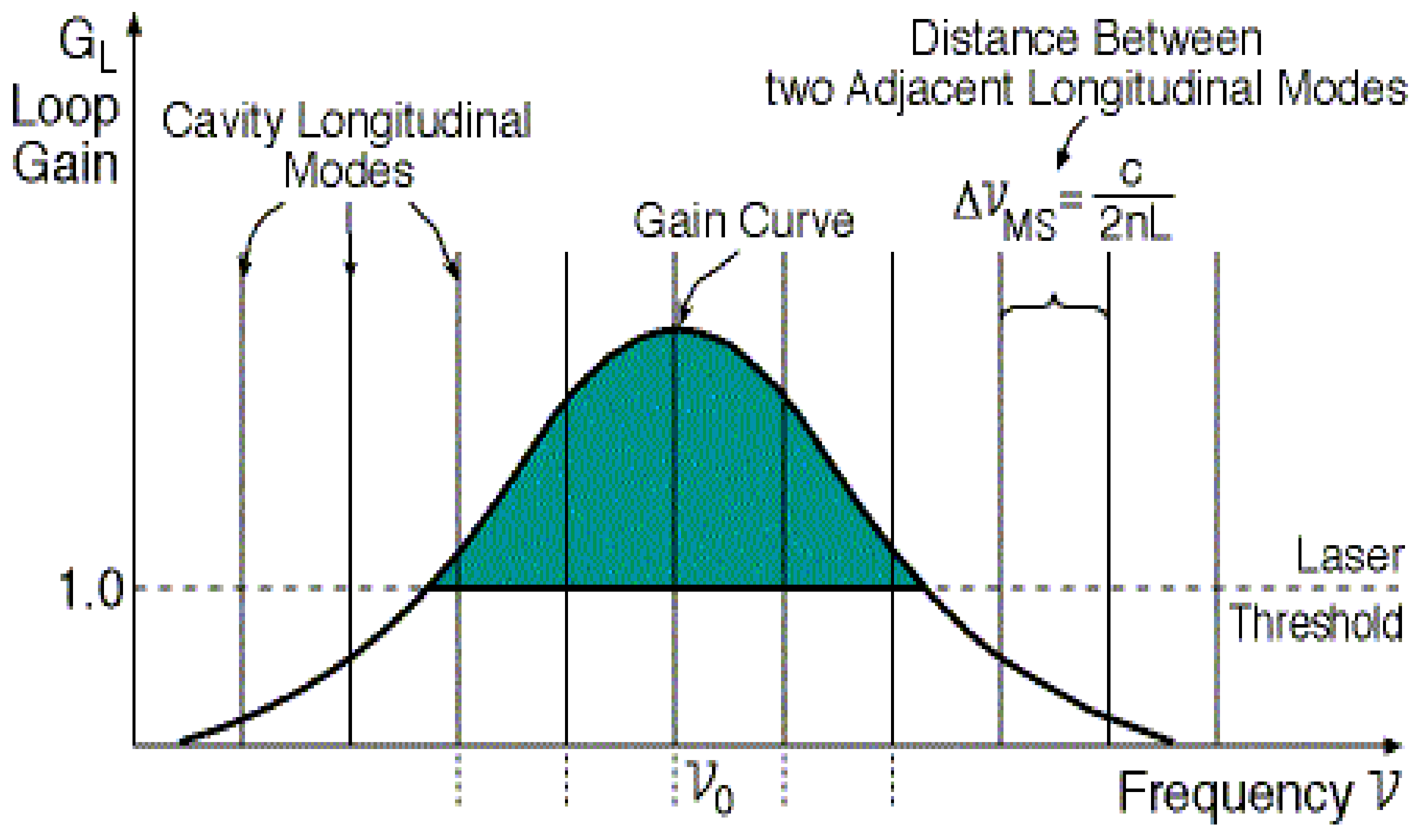
Fluorescence line is described by plotting spontaneous emission radiation intensity
as a function of frequency
(or wavelength),
for the specific lasing transition.



The main transition in the visible spectrum is from level E_5 to level E_2, and the emission is at **red wavelength of 632.8 [nm]**.

Gain Curve of the Active Medium

The gain curve of the active mediumis marked with the **lasing threshold** and **possible longitudinal modes of the laser.**



Longitudinal Modes in a Laser explains that **only specific frequencies are possible inside the optical cavity of a laser**, according to **standing wave condition**.

From all these possible frequencies, only those that have **amplification above certain minimum, to overcome,** will be emitted out of the laser.
This minimum amplification is defined as **lasing threshold**.
The condition of minimum amplification means that the amplification is equal to losses, so that in a round trip path inside the cavity $\mathbf{G_L = 1}$.

The height of each lasing line depends on the losses in a round trip inside the cavity, including the emitted radiation through the output coupler.

The marked region under the curve and above the lasing threshold include the range where lasing can occur.

The height of the gain curve depend on the length of the active medium and its excitation.

The **possible longitudinal modes** of the laser are marked as perpendicular lines at equal distances from each other.
Only frequencies from those allowed inside the cavity, are above the lasing threshold.

The Number of Longitudinal Optical Modes

In this laser 5 frequencies are allowed at the output, and they are spaced at equal distances, which are equal to the mode spacing:

$$\Delta\nu_{MS} = \frac{c}{2\ nL}$$

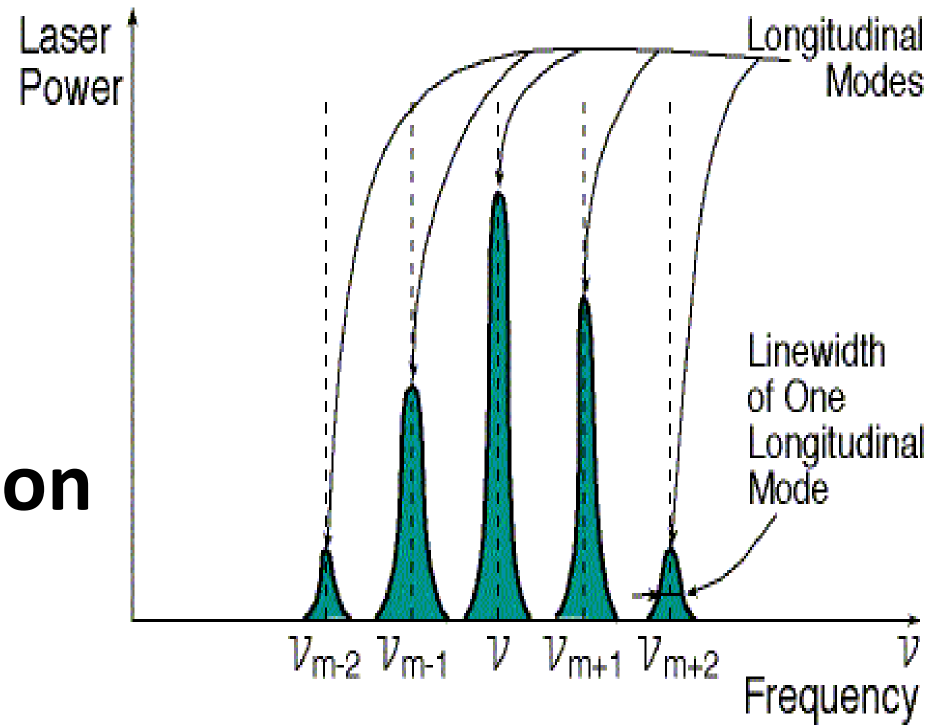


Figure: Spectral distribution of laser lines.

Loop Gain

Contrary to amplifying the radiation, there are many **losses**:

- Scattering and absorption losses at the end mirrors.
- Output radiation through the output coupler.
- Scattering and absorption losses in the active medium, and at the side walls of the laser.
- Diffraction losses because of the finite size of the laser components.

These losses cause some of the radiation not to take part the lasing process.

A necessary condition for lasing is that the total gain will be a little higher than all the losses.

Loop Gain (G_L)

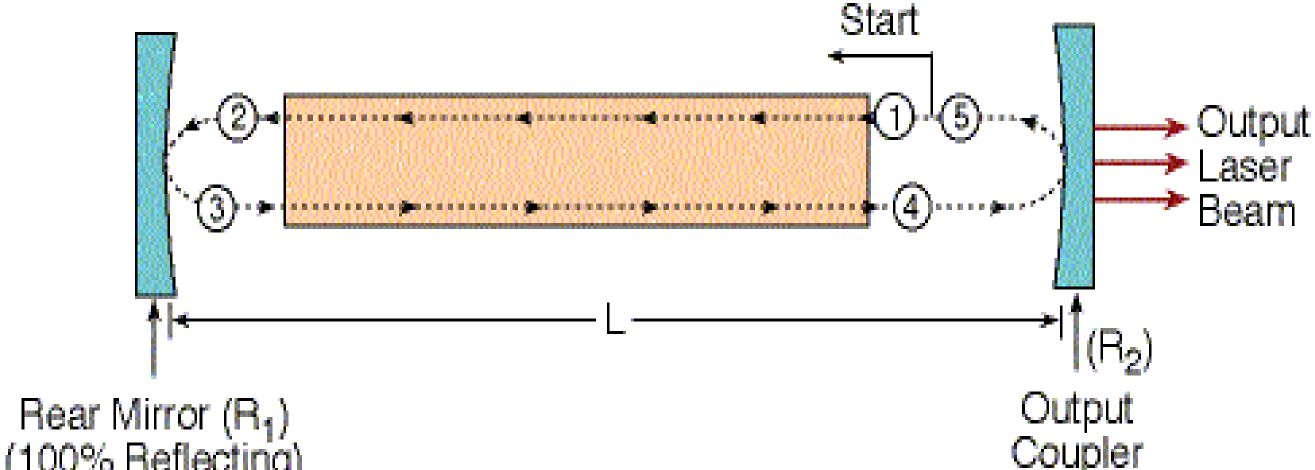
The round trip path of the radiation through the laser cavity: The path is divided to sections numbered by 1-5, while point same point as “1”. By definition, **Loop Gain** is given by:

$$G_L = E_5/E_1$$

G_L = Loop Gain.

E =$_1$ Intensity of radiation at the beginning of the loop.

E_5 = Intensity of radiation at the end of the loop.



Calculating Loop Gain (G_L) Without Losses

G_A = **Active medium gain**

$E_2 = G_A * E_1$

length of the cavity, such that the active medium feel the length of the laser cavity.

On the way from point "2" to point "3", As a result: $E_3 = R_1 * G_A * E_1$

through the active medium, and amplified. Thus: $E_4 = R_1 * G_A^2 * E_1$

the output coupler, which have a reflectivity R_2. Thus: $E_5 = R_1 * R_2 * G_A^2 * E_1$

This completes the loop.

Calculating Loop Gain (G_L) With Losses

We **assume that the losses occur uniformly along the length cavity (L)**.

In analogy to the Lambert formula for losses , we define **loss coefficient (a)**, **absorption factor M**: **M = exp(-2aL)**

a= **Loss coefficient**

2L = **Path Length**, which is twice the length of the cavity.

Adding the **loss factor** **(M)** to the equation of E_5:

$$E_5 = R_1 * R_2 * G_A^2 * E_1 * M$$

From this we can calculate the **Loop gain**:

$$G_L = E_5/E_1 = R_1 * R_2 * G_A^2 * M$$

Calculating Gain Threshold $(G_L)_{th}$

As we assumed uniform distribution of the **loss coefficient (a)**,
we now define **gain coefficient (b)**, and assume **active medium gain (G_A)** as distributed uniformly along the length of the cavity.

$$G_A = \exp(+bL)$$

Substituting the last equation in the Loop Gain:

$$G_L = R_1 * R_2 * \exp(2(b-a)L))$$

Conclusion:

There is a threshold condition for amplification, in order to create oscillation inside the laser. This Threshold Gain is marked with index **"th"**.

For continuous laser , the threshold condition is:

$$(G_L)_{th} = 1 = R_1\, R_2\, G_A\, M = R_1 * R_2 * \exp(2(b-a)L)$$

Mathematical Expressions of fluorescence linewidth

Fluorescence linewidth is expressed by wavelengths, or frequencies, of two points on the spontaneous emission graph at **half the maximum height**.

$$\Delta\nu = |\nu_2 - \nu_1| = \left|\frac{c}{\lambda_2} - \frac{c}{\lambda_1}\right| = \left|\frac{c\lambda_1 - c\lambda_2}{\lambda_1\lambda_2}\right| = \frac{c\Delta\lambda}{\lambda_1\lambda_2}$$

The linewidth (D l) is much smaller than each of the wavelengths (Dl<<l_1,l_2).

Thus the **approximation:** l_1*l_2 = can be used.

l_0 = Wavelength at the center of emission spectrum of the laser.

The result is:

In a similar way:

these mathematical relations will be used for determining the **coherence** of the laser.

Broadening the Fluorescence line

Certain mechanisms are responsible for broadening the linewidth of a laser:

1. **Natural broadening.**
2. **Doppler Broadening.**
3. **Pressure broadening.**

For many applications, especially when **temporal coherence** is required a **small linewidth of the emitted laser wavelength is required**.

Natural broadening.

This broadening is always present, and comes from the **finite transition time from the upper laser level to the lower laser level.**

Natural linewidth is narrow: 10^4 - 10^8 [Hz], compared to the radiation frequency of visible light: 10^{14} [Hz].

Each energy level has a specific width (Dn), and specific lifetime (Dt).
Natural broadening results from the **Heisenberg uncertainty principle:**

DE*Dt > h

DE = h*

Dn Dn > 1/

D**t**

Numerical examples:

Dt = 10^{-8} [s] = = > Dn = 10^8 [Hz]

Dt = 10^{-4} [s] = = > Dn = 10^4 [Hz]

The longer the specific energy level transition lifetime, the narrower is its linewidth.

Doppler Broadening

Doppler shift is a well known phenomena in wave motion.

It occurs when the source is in relative motion to the receiver.

The frequency detected is shifted by an amount determined by the **relative velocity between the source and the receiver.**

Since **gas molecules are in constant motion in random directions**, each molecule emit light while it is moving relative to the laser axis in a different direction. These distribution of frequency shifts cause the broadening of the laser linewidth.

Doppler broadening occur especially in **gas lasers**, as a result of movement of gas molecules.

Its influence is mostly in **low pressure gas lasers**

Pressure (collisions) broadening

It is caused by collisions between the molecules of the gas.
Pressure broadening is the **largest broadening mechanism in gas lasers** with pressure of more than 10 mm Hg.
As the pressure increase, the broadening increase.

At constant pressure (P), as the temperature (T) increases:

PV = nRT
P = const = = > V increases when T increases.
Since the Volume (V) increases, the number of collisions decrease. Thus, pressure ((collisions) broadening decrease.

Numerical example:

1. At room temperature, the linewidth of CO_2 laser with gas pressure of 10 [torr] is 55 [MHz].
2. At room temperature, the linewidth of CO_2 laser with gas pressure of 100 [torr] is 500 [MHz].
3. Above 100 [torr], the increase rate of broadening is about 6.5 [MHz] for each increase in pressure of 1 [torr].

Linewidth broadening

the result of broadening of the fluorescence linewidth.