introduction

Signal: is a piece of information explained a function of time for example (voice, video)

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system: is physical or mathmatical (hardware, soft ware) that perform opration on signal to extract or modify information for example (law pass filter) → is system to remove ahigh frequencies from signal

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DSP: is a sequence of values defined at specific time intervals, represented mathematicly as x(n), where (n) is an integer and this signal taked from analog signal

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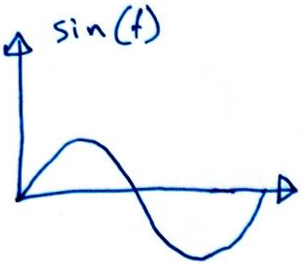
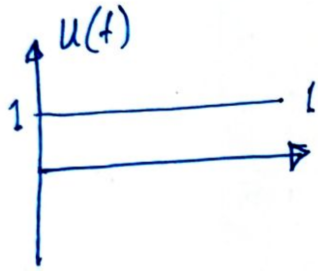
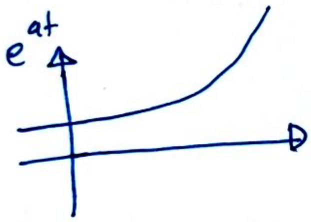
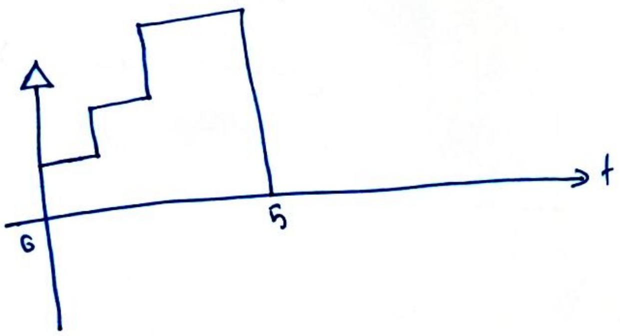
Basic elements of DSP

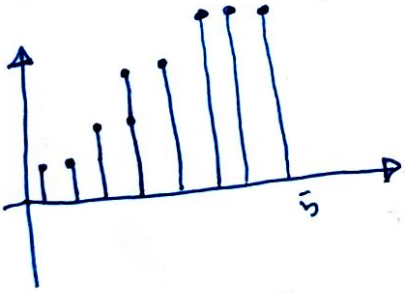
Analog → PrF → ADC → DSP → DAC → PoF → Analoge di'





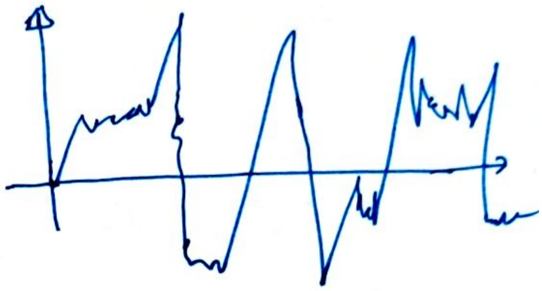






random

noise



P

analog	Digital
	
time continuous time	discrete time
* تتعامل مع زمن مستمر	* لا تتعامل مع زمن مستمر
time = 1, 2, 3, −10, 1.5, −2.5	n = 0, 1, 2, 5, −9, −10 * يجب أن يكون عدد صحيح

* classification of signals

① continuous time or discret time

continuous time

discret. time

② Determinstic or random
محسوبة غير محسوبة

$$\left(\begin{array}{l} \sin(t) \\ u(t) \\ \cos(t) \\ e^{at} \end{array} \right)$$

most important digital time signal

① unit step

② unit sample or implse

③ unit ramp

④ exponatial

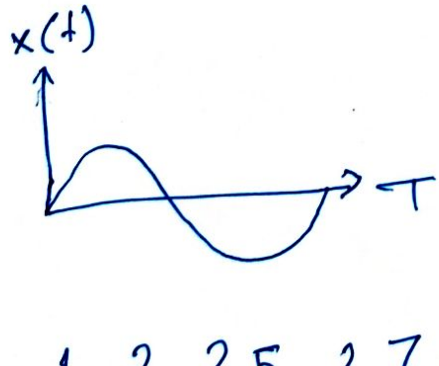
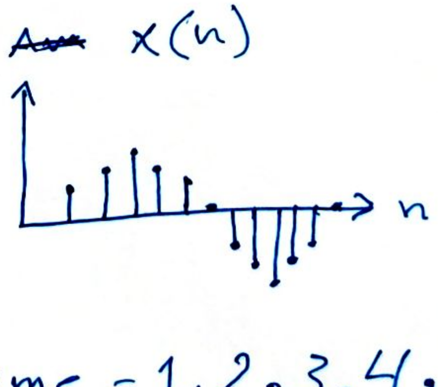
⑤ cos function $\longrightarrow$ sinusoidal sequence

$$x(n) = \cos(wn) \Rightarrow x(n) = \cos(nw)$$

$w \rightarrow$ rad/sample

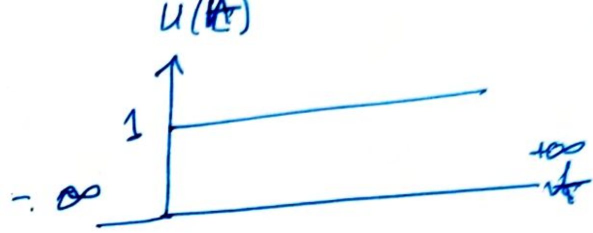
$$f = \frac{w}{2\pi} \qquad f = \frac{1}{T} \qquad w = 2\pi f$$

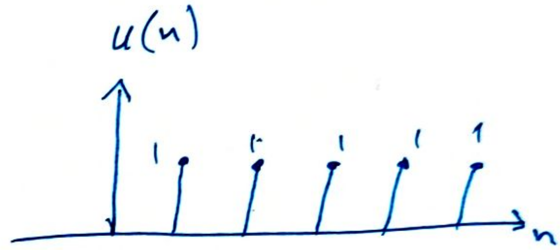
$$w = \frac{2\pi}{\text{number of sequen}}$$

analog	digital or discrete
	
time = 1, 2, 2.5, 2.7, 3, -2	time = 1, 2, 3, 4, 5, 6 ...

① unit step

$$u(t) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases}$$





Ex: sketch the sequence

① $x(n) = 3u(n-1)$

② $x(n) = -2\,u(n+1)$

③ $x(n) = u(n+2)$ ⟶ H.W

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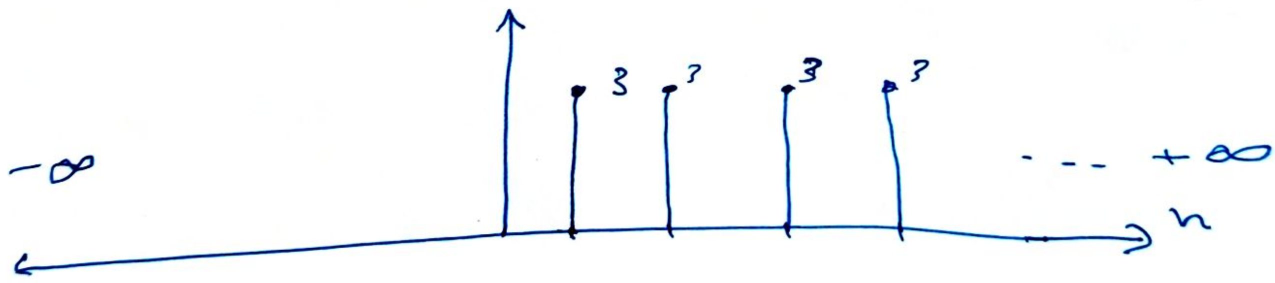
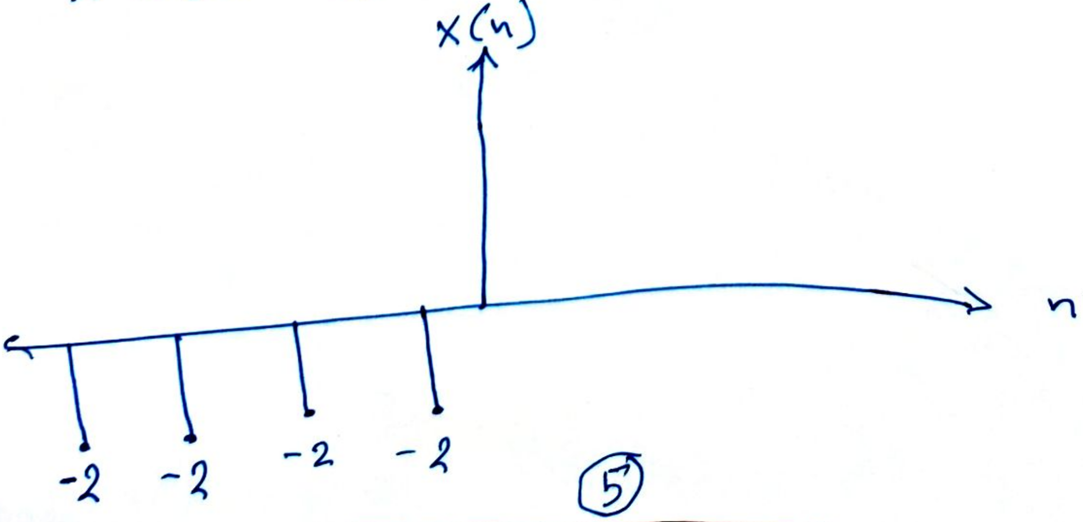
① $x(n) = 3u(n-1)$

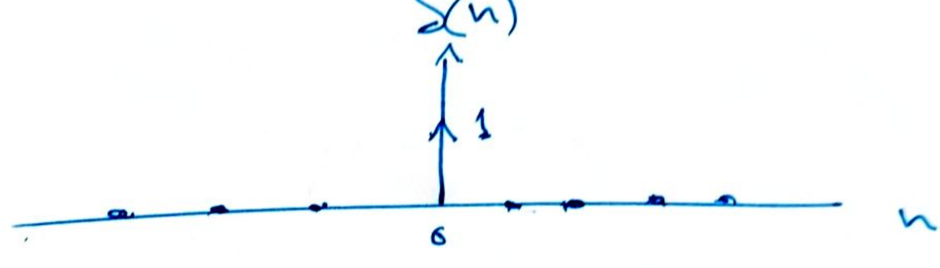


② $x(n) = -2\,u(n+1)$



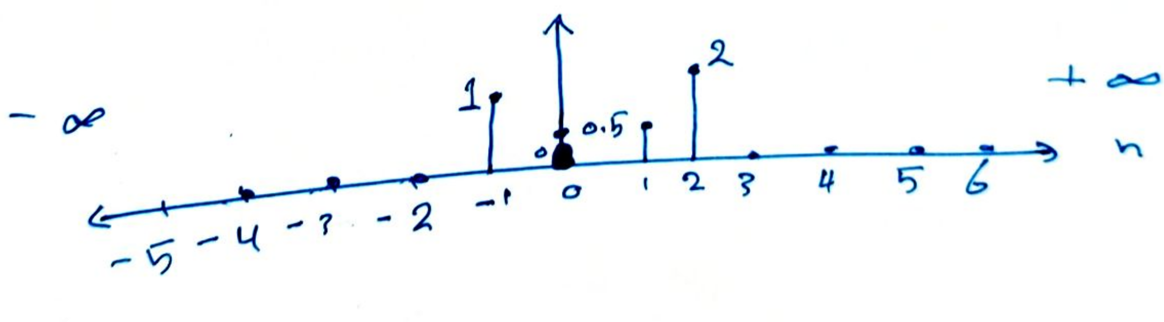
② unit impuls or unit sample

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases}$$



Ex: sketch the sequence

$$x(n) = \delta(n+1) + 0.5\,\delta(n-1) + 2\,\delta(n-2)$$



H.W

① $x(n) = \delta(n) + 3\,\delta(n-1) + 2\,\delta(n-3)$

② $x(n) = 3\,\delta(n+2) + 0.5\,\delta(n) + 2\,\delta(n-1)$

③ Sinusoidal sequence

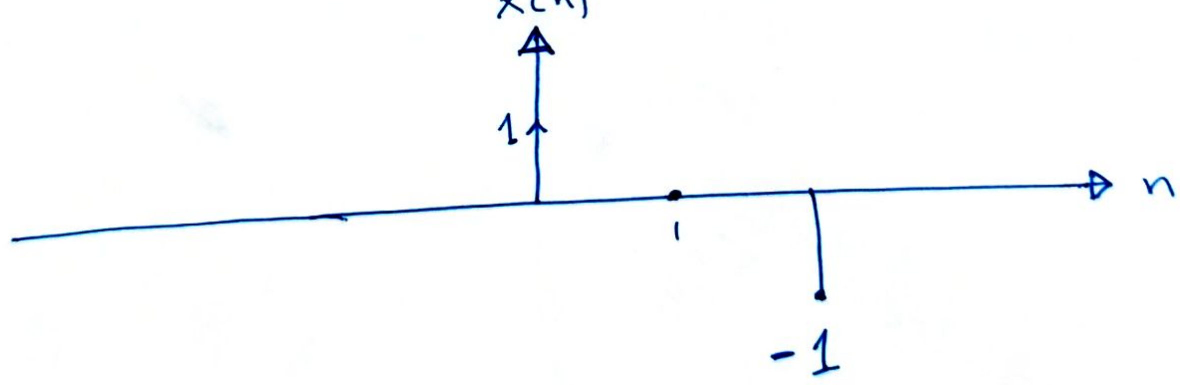
$$x(n) = \cos wn \qquad 0 \leq n$$

Ex: $\cos\left(\frac{n\pi}{2}\right)\underbrace{\left[u(n) - u(n-3)\right]}$

$n = 0, 1, 2$

n	$\cos\left(\frac{n\pi}{2}\right)$
0	$\cos\left(\frac{0\,\pi}{2}\right) = \cos 0 = 1$
1	$\cos\left(\frac{\pi}{2}\right) = \cos\frac{\pi}{2} = 0$
2	$\cos\frac{\cancel{2}\pi}{\cancel{2}} = \cos\pi = -1$

$x(n) =$

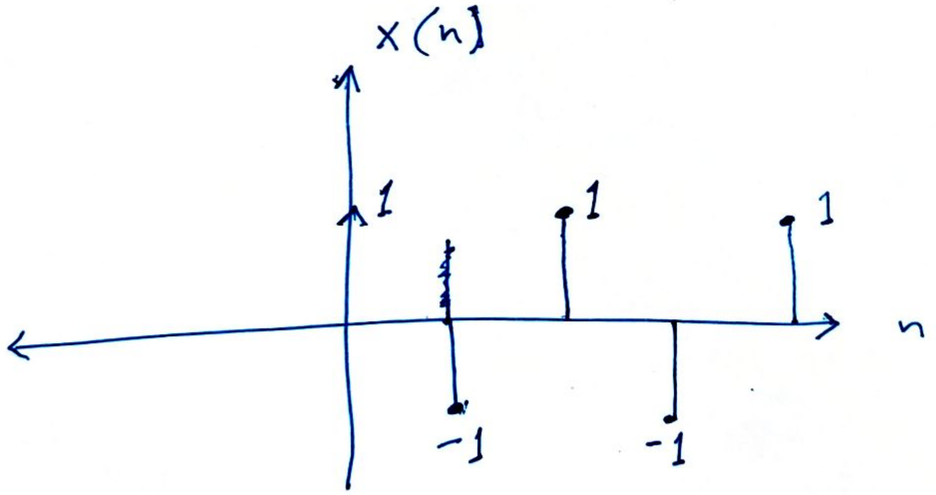


Ex: $\cos n\pi \; u(n)$

sol

n	$\cos n\pi$
0	$\cos 0\pi = \cos 0 = 1$
1	$\cos \pi = -1$
2	$\cos 2\pi = 1$
3	$\cos 3\pi = -1$
4	$\cos 4\pi = 1$

$$x(n) = \{ \underset{\uparrow}{1}, -1, 1, -1, 1 \}$$



H.W

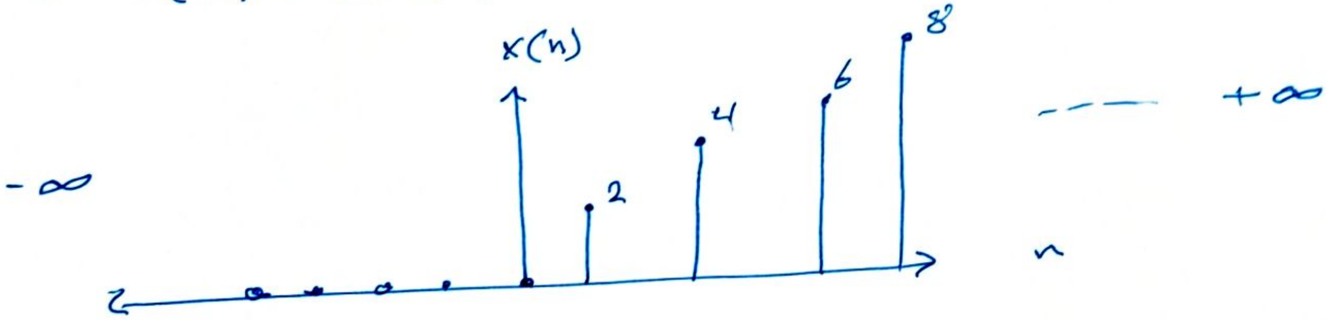
1 Ex: $\sin \frac{n\pi}{2} \; u(n)$ HW

2 Ex: $n\,[u(n) - u(n-3)]$

④ Unit ramp

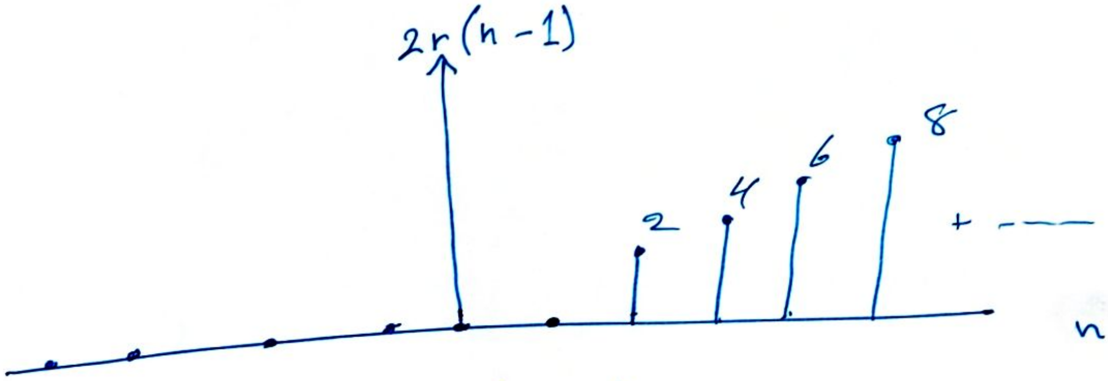
$$r(n) = \begin{cases} n & n \geq \\ 0 & n < 0 \end{cases}$$

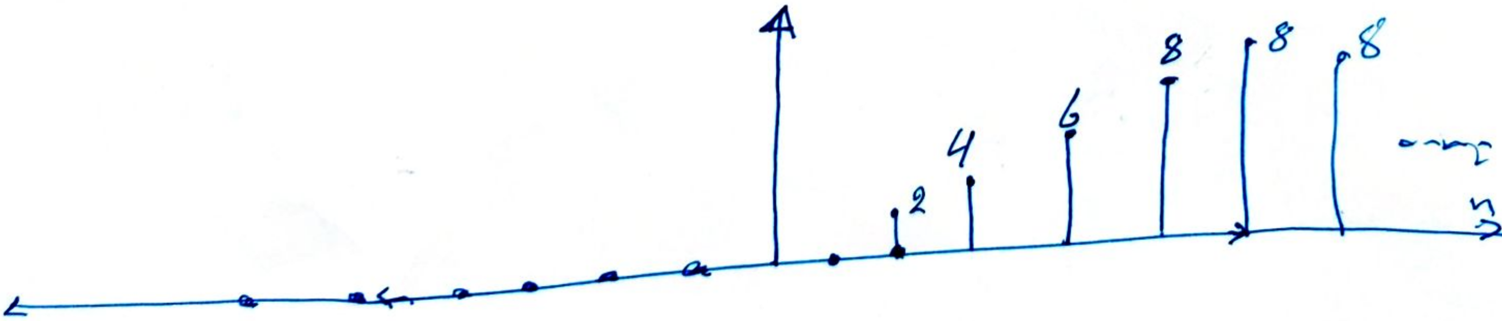
Ex ÷ $x(n) = 2r(n)$



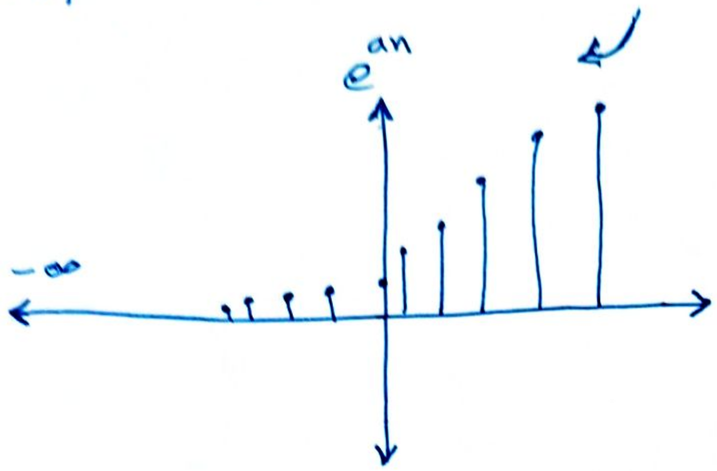
Ex ÷ $x(n) = 2r(n-1) - 2r(n-5)$

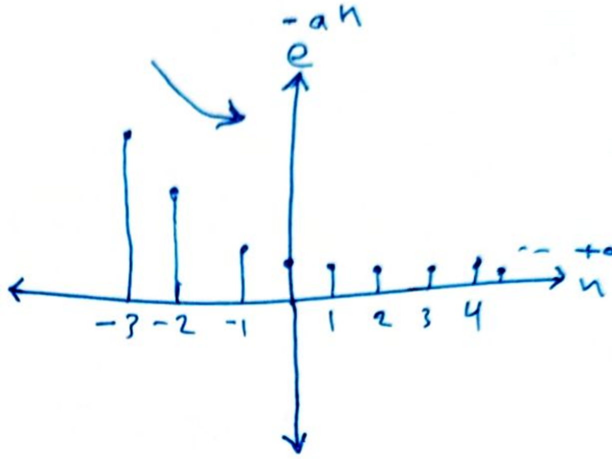
Sol



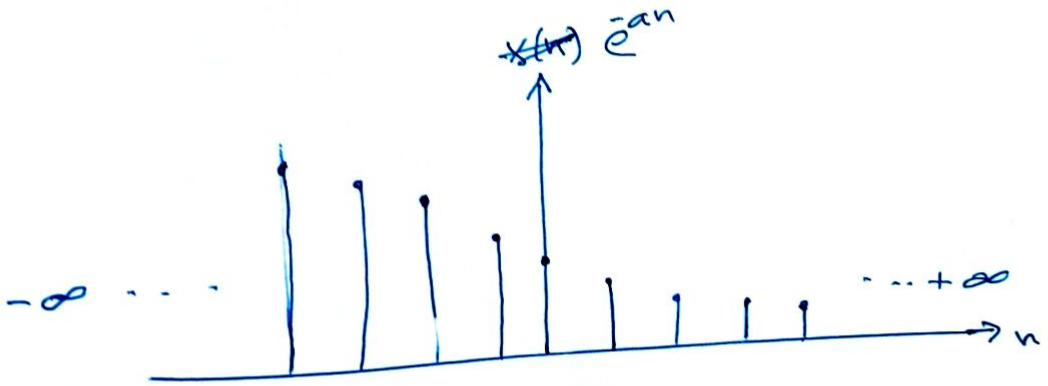


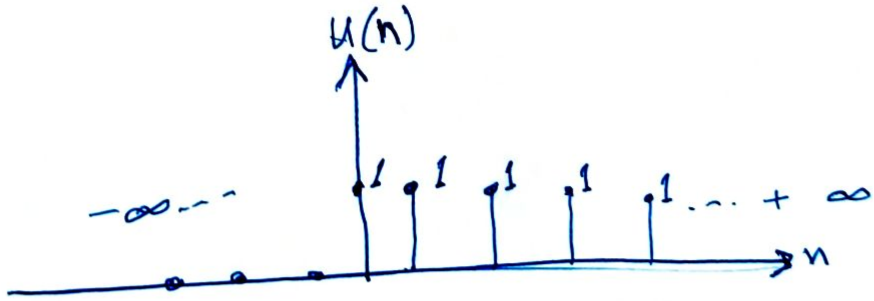
⑤ exponatial

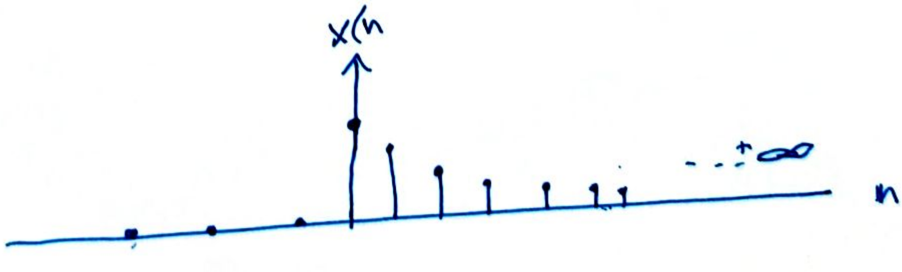




Ex÷ $x(n) = e^{-an}\, u(n)$







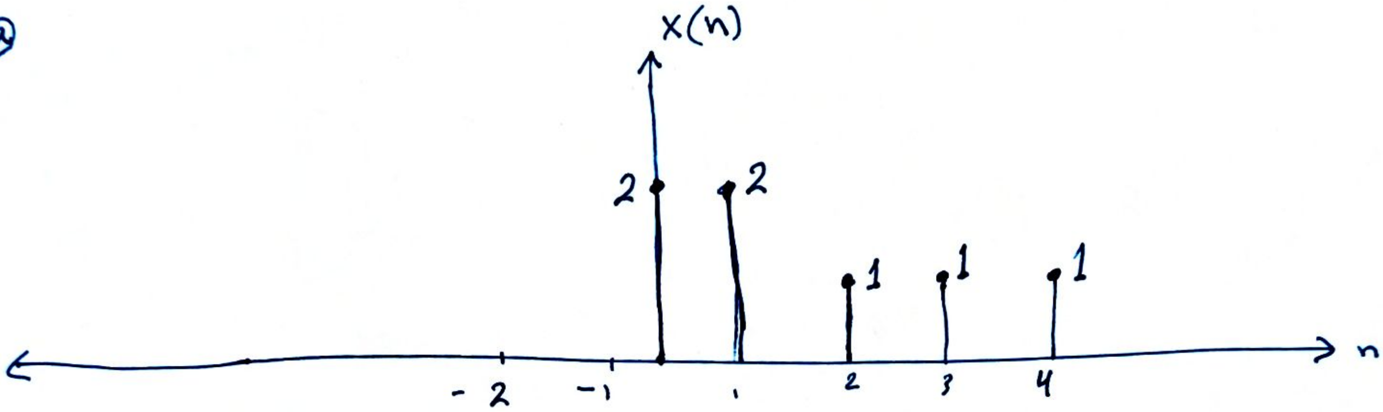
EX : given the sequence

$$x(n) = 2\delta(n) + 2\delta(n-1) + \delta(n-2) + \delta(n-3) + \delta(n-4)$$

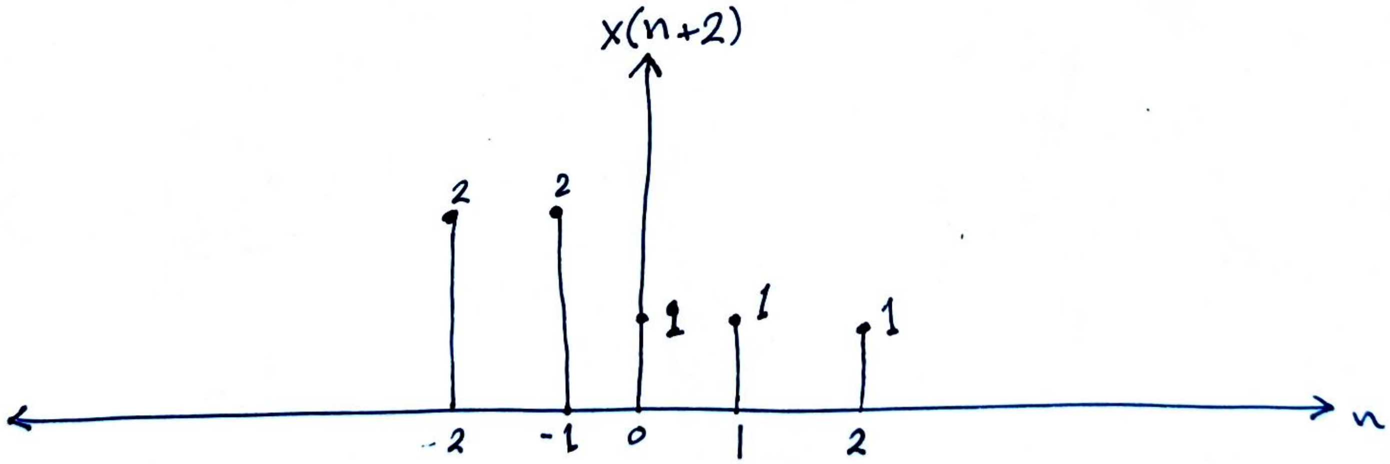
make a sketch of :

(a) $x(n)$ (b) $x(n+2)$ (d) $x(-n+2)$

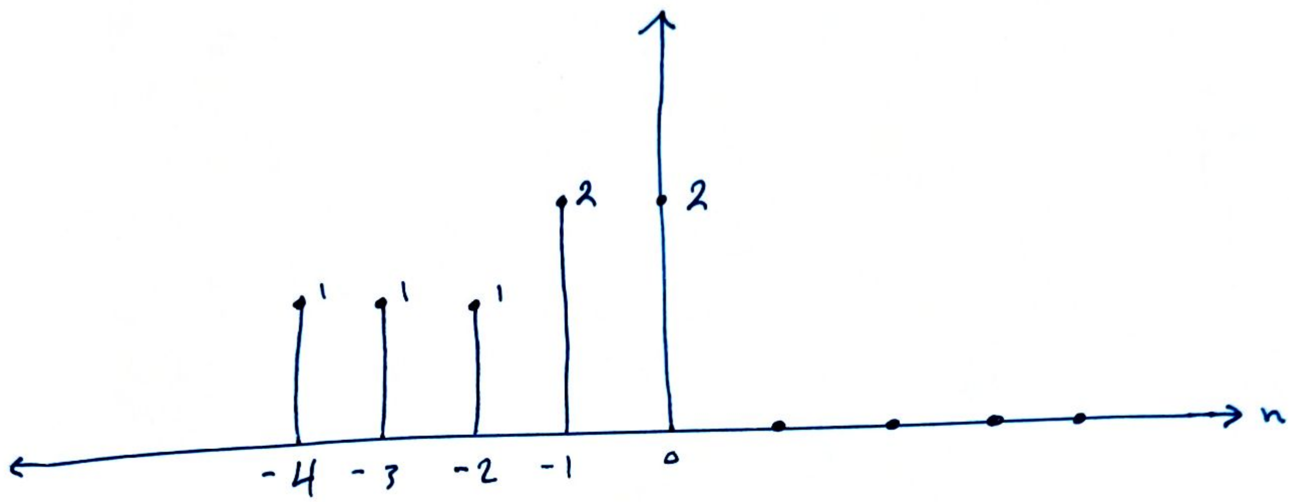
(a)



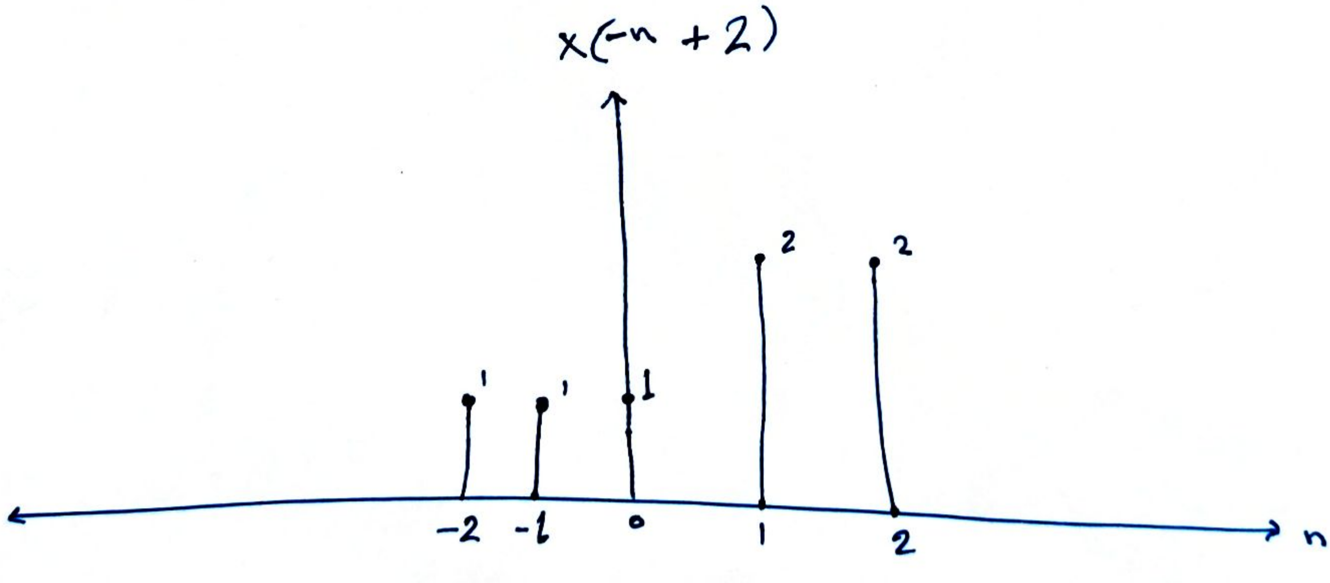
(b)



ⓒ $x(-n)$



ⓓ $x(-n+2)$

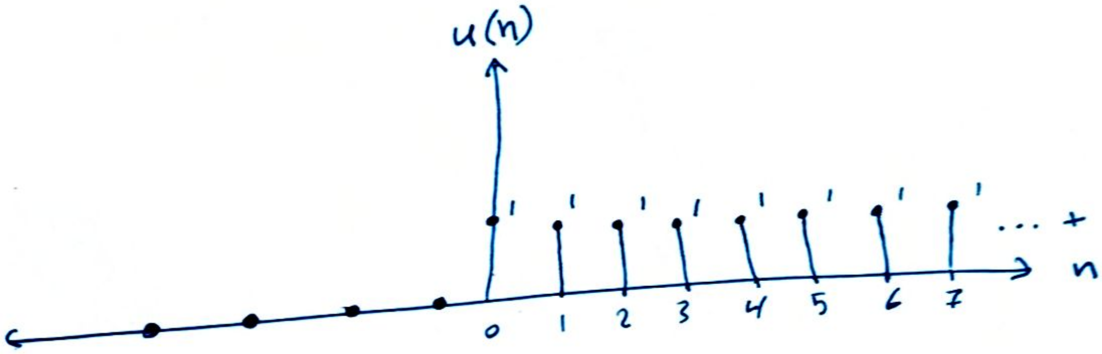


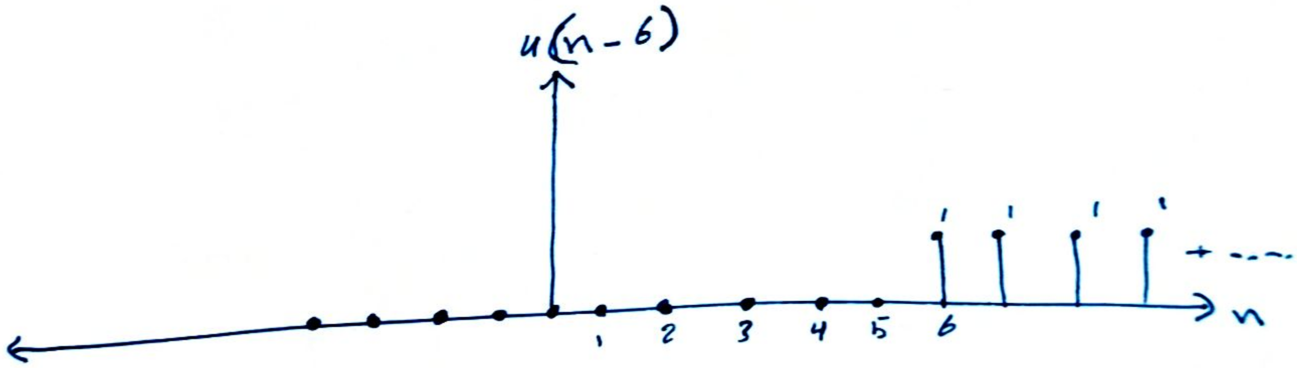
Ex:

$$x(n) = (6-n)\,[u(n) - u(n-6)]$$

① $x(n-3)$ ② $3x(n+1)$ ③ $x(-n)$

④ $x(4-n)$ ⑤ $x(2n-3)$ ⑥ $x(8-3n)$ ⑦ $x(n^2-2n+1)$





$[u(n) - u(n-6)]$

Ex:- consider the Length 7 sequences defined for $-3 \leq n \leq 3$

$x(n) = \{3, -2, 0, 1, 4, 5, 2\}$

$y(n) = \{0, 7, 1, -3, 4, 9, 2\}$

$w(n) = \{-5, 4, 3, 6, -5, 0, 1\}$

Genrate the following sequences

ⓐ $u(n) = x(n) + y(n)$

ⓑ $v(n) = x(n) \cdot w(n)$

ⓒ $S(n) = y(n) - w(n)$

ⓓ $r(n) = 4.5\, y(n)$

Sol

ⓐ $u(n) = x(n) + y(n)$

$= \{3, -2, 0, 1, 4, 5, 2\} + \{0, 7, 1, -3, 4, 9, 2\}$

$u(n) = \{3, 5, 1, -2, 8, 14, 4\}$

ⓑ $v(n) = x(n) \cdot w(n)$

$v(n) = \{3, -2, 0, 1, 4, 5, 2\} \cdot \{-5, 4, 3, 6, -5, 0, 1\}$

$v(n) = \{-15, -8, 0, 6, -20, 0, 2\}$

ⓒ and ⓓ $\longrightarrow$ H.W

Ex: Represent the following sequence

(a) Graphical representation

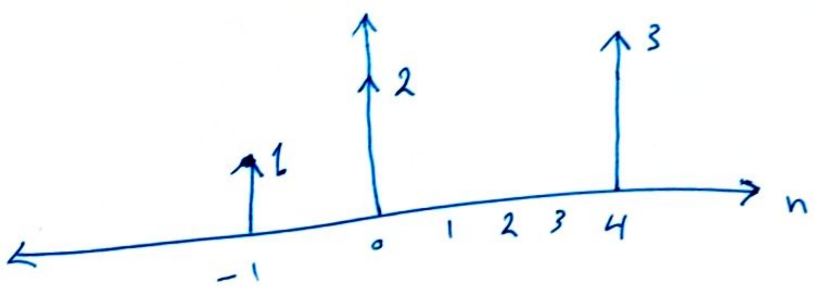
(b) Function representation.

(a) $x(n) = \delta(n+1) + 2\delta(n) + 3\delta(n-4)$

(b) $x(n) = (n+1)\,[u(n) - u(n-3)]$

Sol:

(a) $x(n) = \delta(n+1) + 2\delta(n) + 3\delta(n-4)$



(b) $x(n) = (n+1)\,[u(n) - u(n-3)]$

from $[u(n) - u(n-3)]$ we conclude

$n = \{0, 1, 2\}$

$\therefore$

n	$n+1$
0	$0+1 = 1$
1	$1+1 = 2$
2	$2+1 = 3$

these are sequence

* these are Amplitude not sequence

HW :

1. $x(n) = 2r(n)$
2. $x(n) = u(n) - u(n-5)$
3. $x(n) = u(n) - u(n-2)$
4. $x(n) = 2u(n)$
5. $x(n) = u(n-3)$
6. $x(n) = 7u(n+3)$
7. $x(n) = (n-2)[u(n-3) - u(n-6)]$
8. $x(n) = 3\delta(n) - 2\delta(n-2) + 4\delta(n+4) + 5\delta(n-6)$
9. $x(n) = 2^n[u(n) - u(n-2)]$
10. $x(n) = (n+1)[u(n) - u(n-3)]$
11. $x(n) = \{2, 1, -2, 3, 1\}$

find (a) $x(2n)$ (b) $4x(n)$ (c) $x(2n+1)$

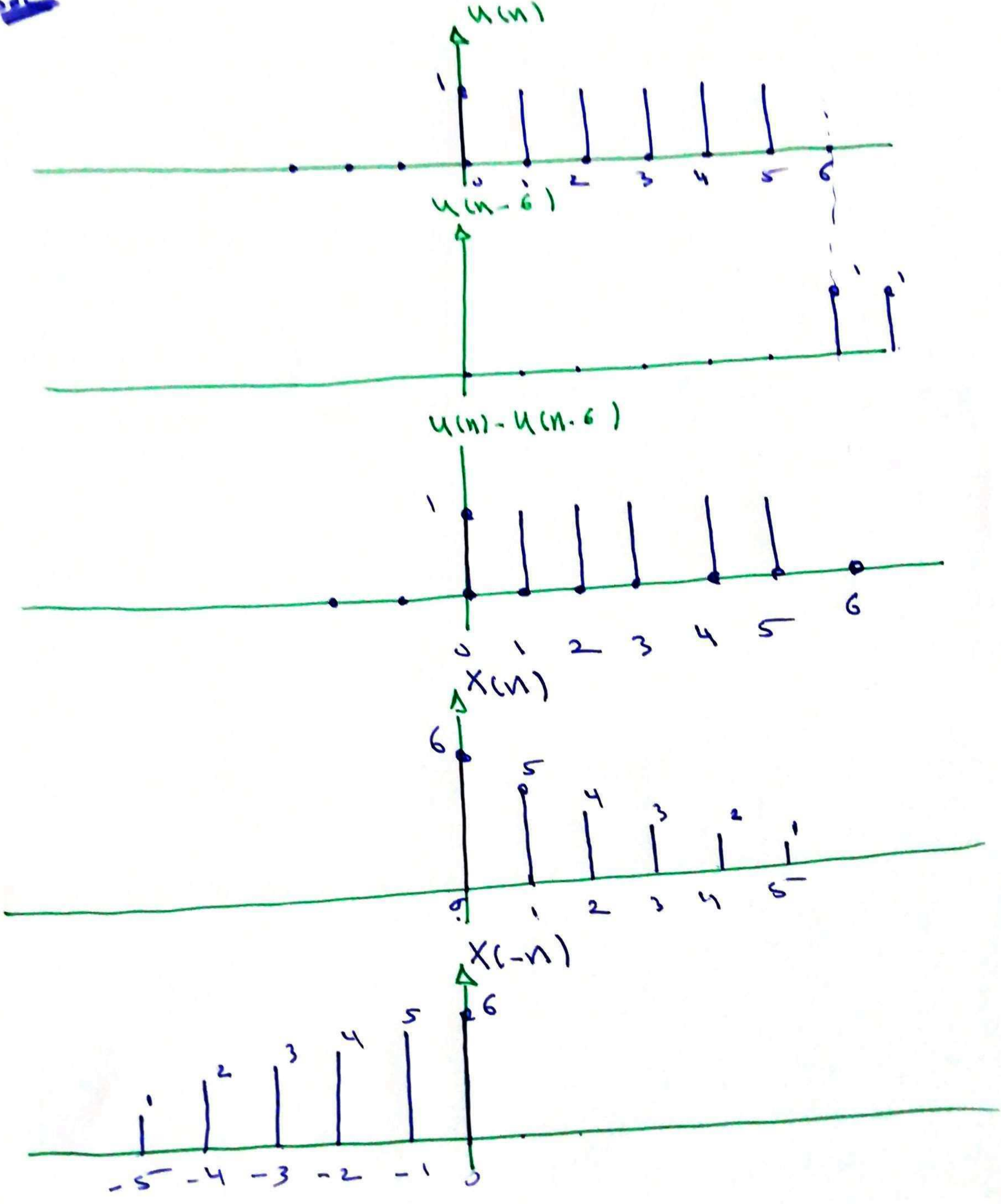
(c) $x(2n+1)$

H.W (3) Let $X(n) = (6-n)\left[U(n) - U(n-6)\right]$

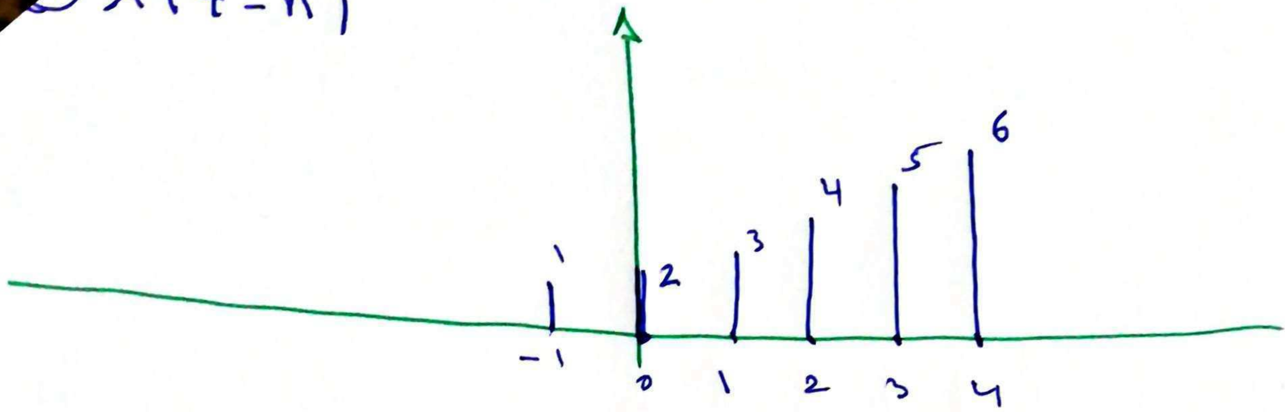
Find ① $X(4-n)$ ② $X(2n-3)$ ③ $X(8-3n)$

④ $X(n^2-2n+1)$

Sol

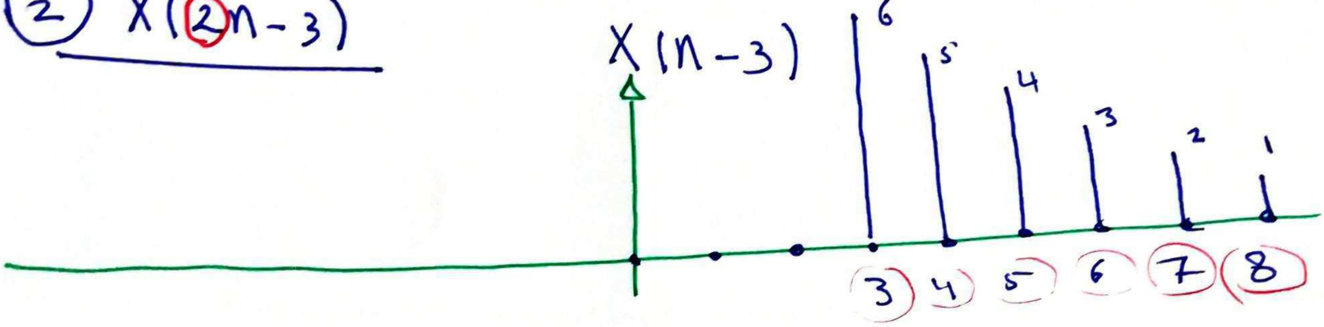


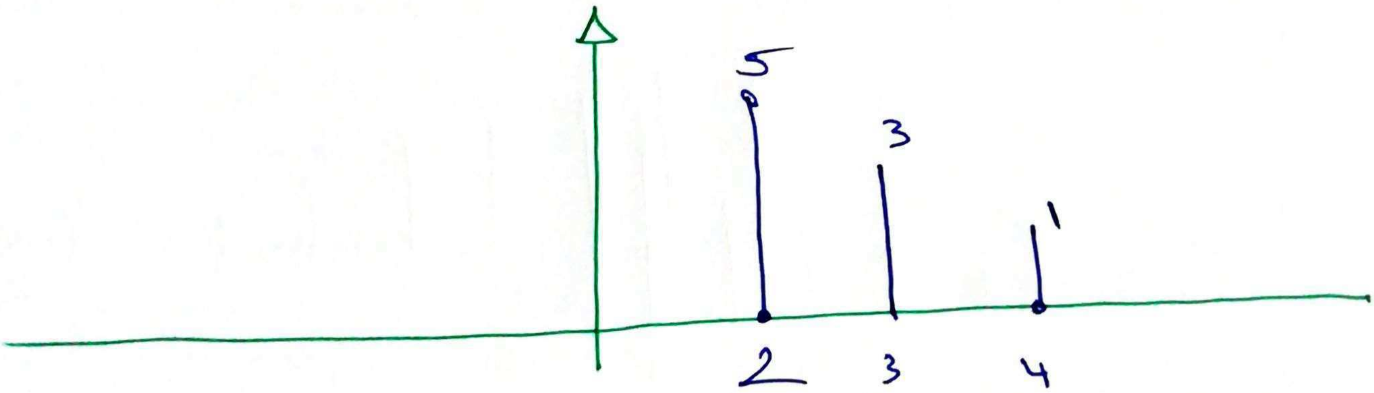
1 $X(4-n)$



2 $X(2n-3)$

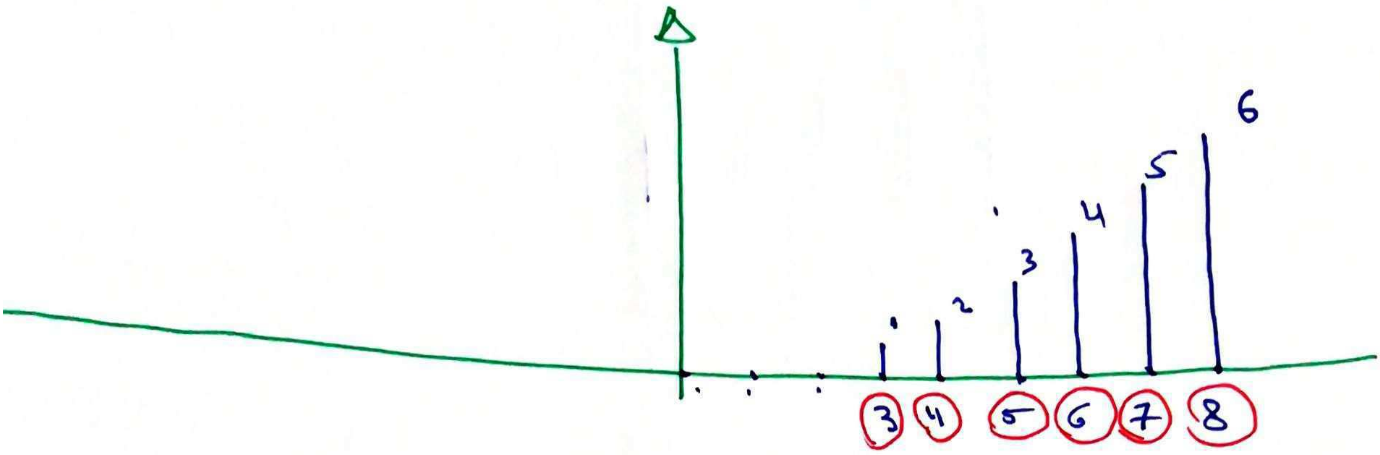
$X(n-3)$

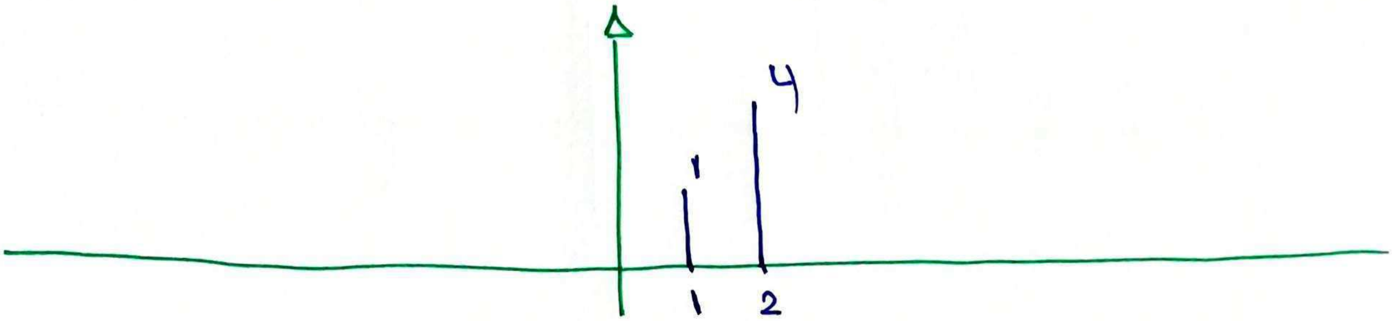




(3) $X(8-3n)$

$X(8-n)$





(4) $X(n^2-2n+1) = X((n-1)(n-1)) = X((n-1)^2)$

$\sqrt{(n-1)^2} = \sqrt{0} \rightarrow n-1=0 \rightarrow n=1$

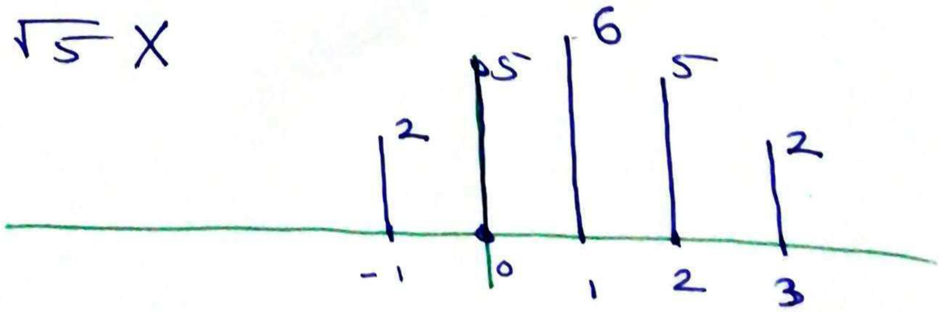
$\sqrt{(n-1)^2} = \sqrt{1} \rightarrow n-1 = \mp 1 \rightarrow n = 1 \mp 1 \begin{cases} \rightarrow n=0 \\ \rightarrow n=2 \end{cases}$

$\sqrt{(n-1)^2} = \sqrt{2} \rightarrow n-1 = \mp\sqrt{2}$ X

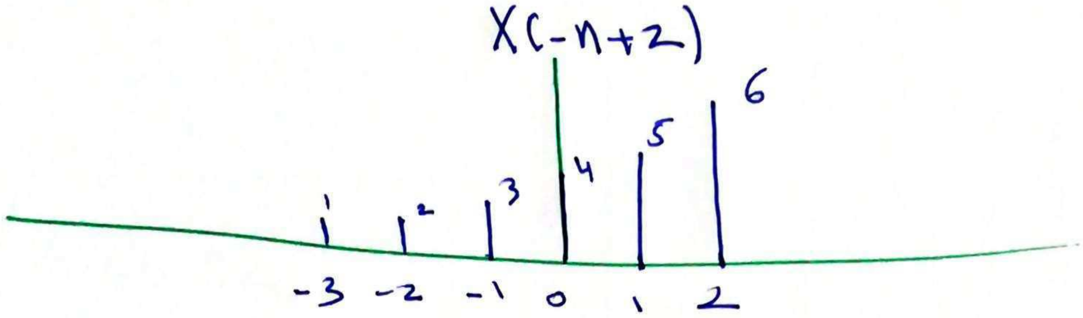
$(n-1)^2 = 3 \rightarrow n-1 = \mp\sqrt{3}$ X

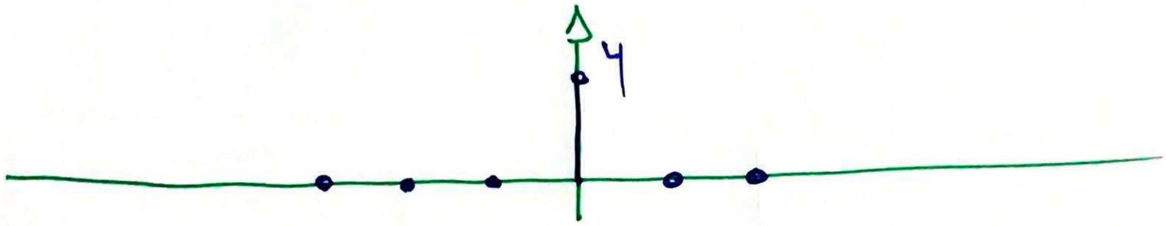
$(n-1)^2 = 4 \rightarrow n-1 = \mp 2 \rightarrow n = 1 \mp 2 \begin{cases} \rightarrow n=-1 \\ \rightarrow n=3 \end{cases}$

$(n-1)^2 = 5 \rightarrow n-1 = \sqrt{5}$ X



$X(-4n+2)$





Ex 7

Consider the system below

$$x(n) \rightarrow \boxed{\text{Delay by 2}} \xrightarrow{x(n-2)} \boxed{\text{Time reversal}} \rightarrow y(n) = x(-n-2)$$

Find $y(n)$ if

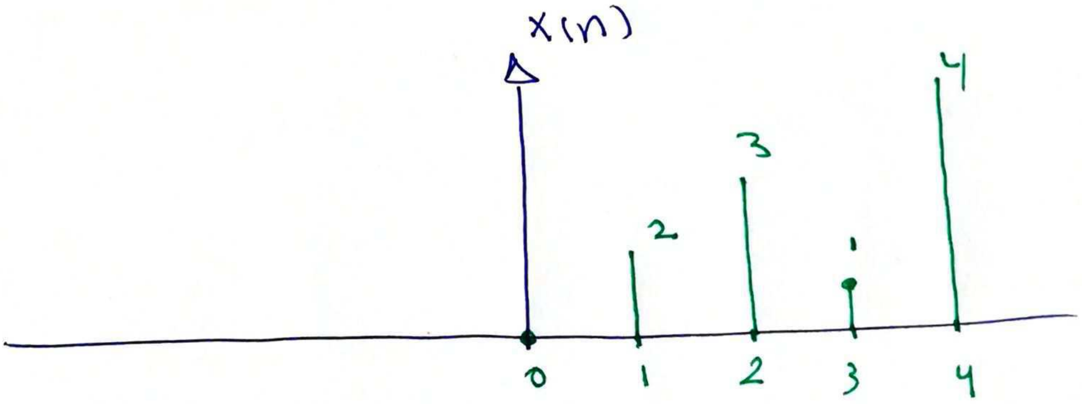
$$x(n) = \begin{array}{c} \begin{array}{ccccc} x(0) & x(1) & x(2) & x(3) & x(4) \end{array} \\ \begin{bmatrix} \underset{\uparrow}{0} & 2 & 3 & 1 & 4 \end{bmatrix} \end{array}$$

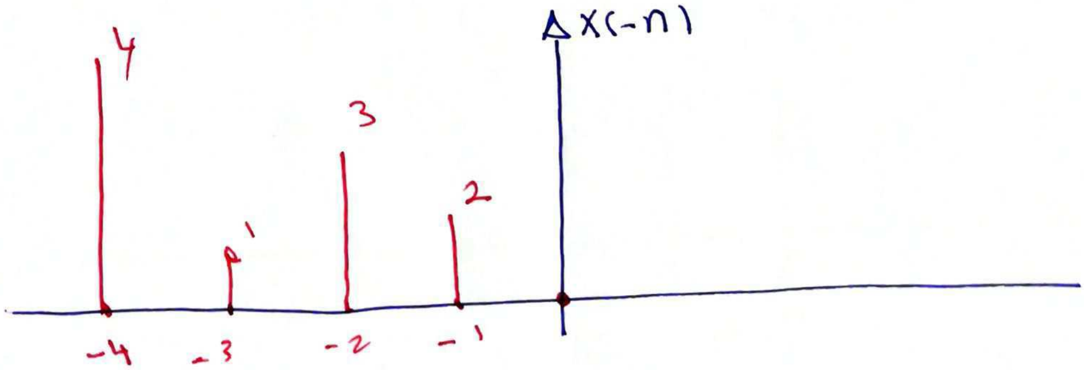
Sol

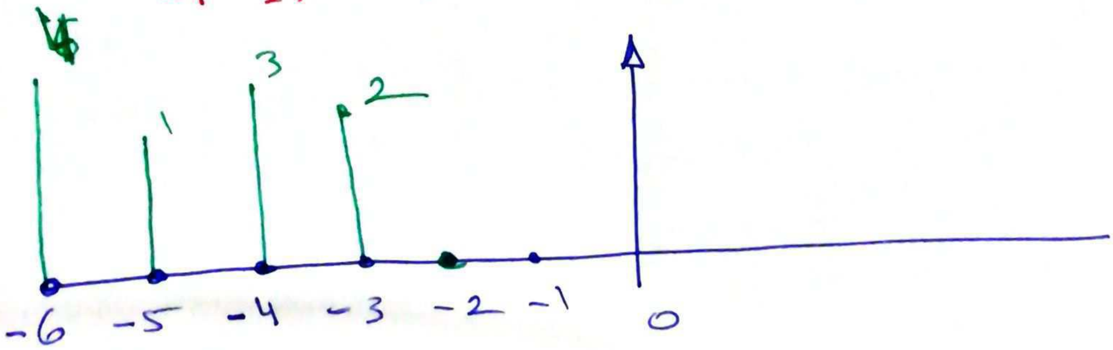
$$x(n) \rightarrow \boxed{\text{Delay}} \rightarrow x(n-1) \qquad \longrightarrow$$

$$x(n) \rightarrow \boxed{\text{advance}} \rightarrow x(n+1) \qquad \longleftarrow$$

$$x(n) \rightarrow \boxed{\text{T.R}} \rightarrow x(-n)$$





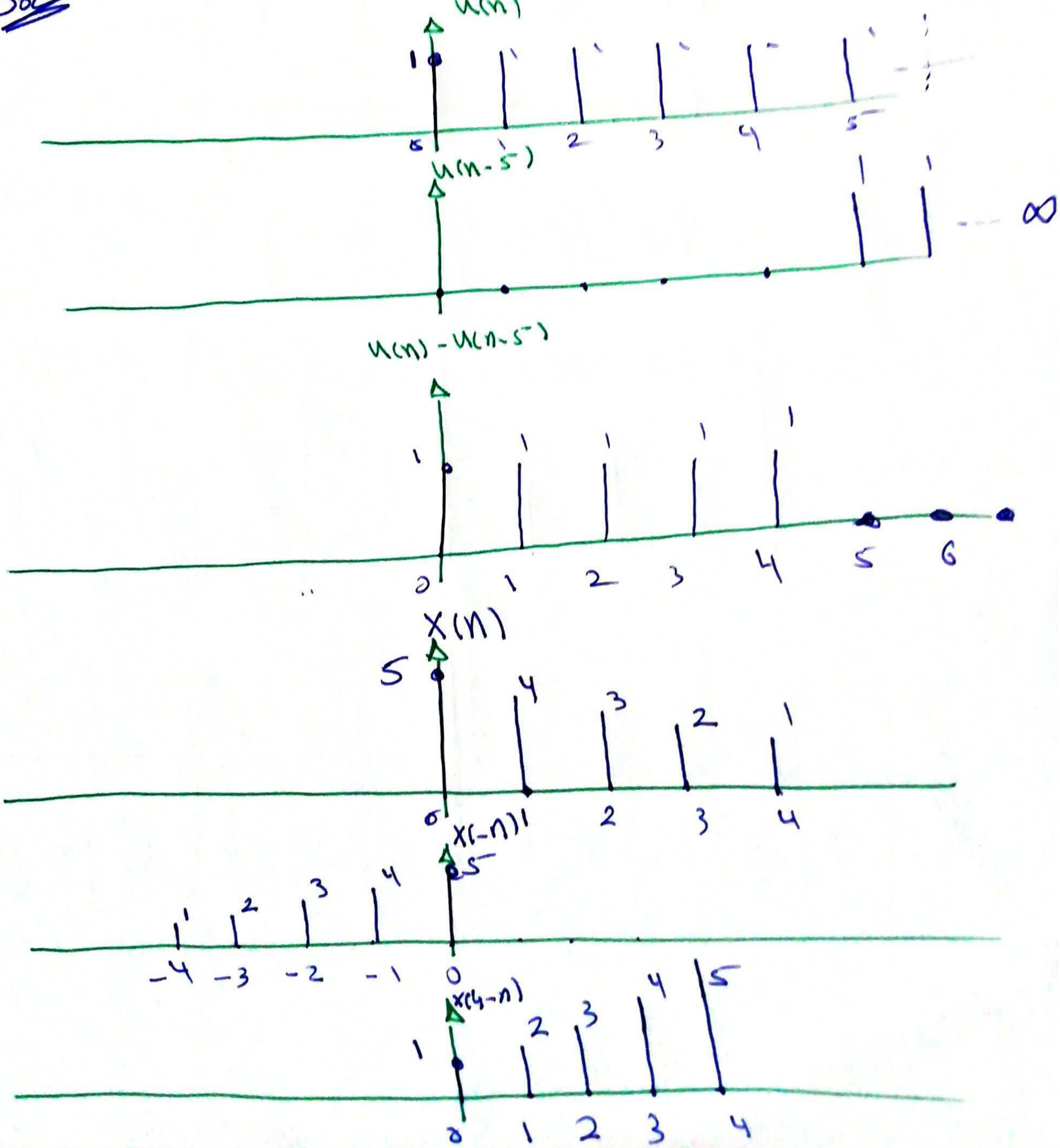


Ex Represent the following Discrete time signal using graphical method.

(a) $X(n) = (5-n)[u(n) - u(n-5)]$

(b) $X(n) \longrightarrow \boxed{T[\cdot]} \longrightarrow Y(n) = X(4-n)$

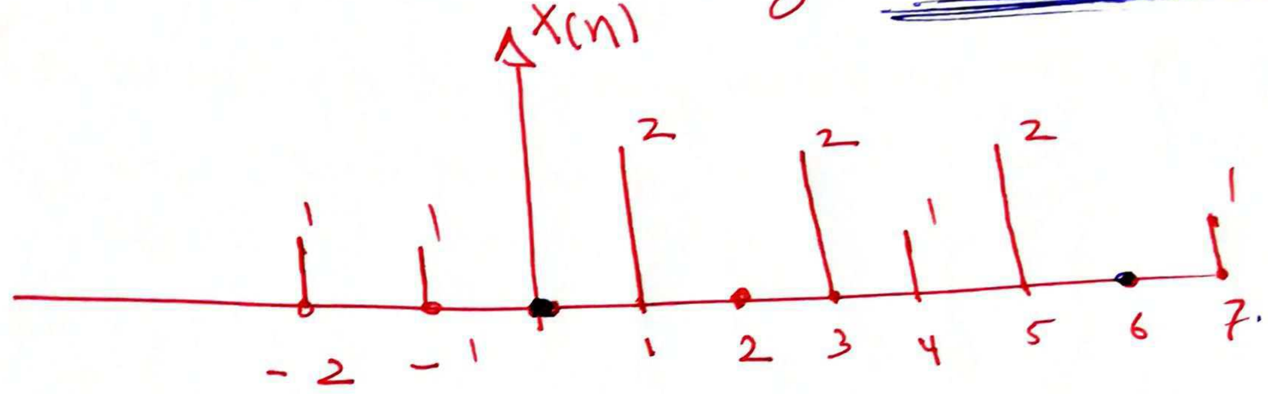
Sol



For a given discrete signal that has been graphical represented as shown in figure below ③

(a) Represent $X(n)$ using unit sample (unit impulse)

(b) Represent $X(n)$ usuing tabular method.



Sol

(a)

$$X(n) = 1\delta(n+2) + 1\delta(n+1) + 2\delta(n-1) + 2\delta(n-3) + 1\delta(n-4) + 2\delta(n-5) + 1\delta(n-7)$$

(b)

n	-2	-1	0	1	2	3	4	5	6	7
$X(n)$	1	1	0	2	0	2	1	2	0	1

For a given discrete time signal represented by the function $X(n)$ as follows: (4)

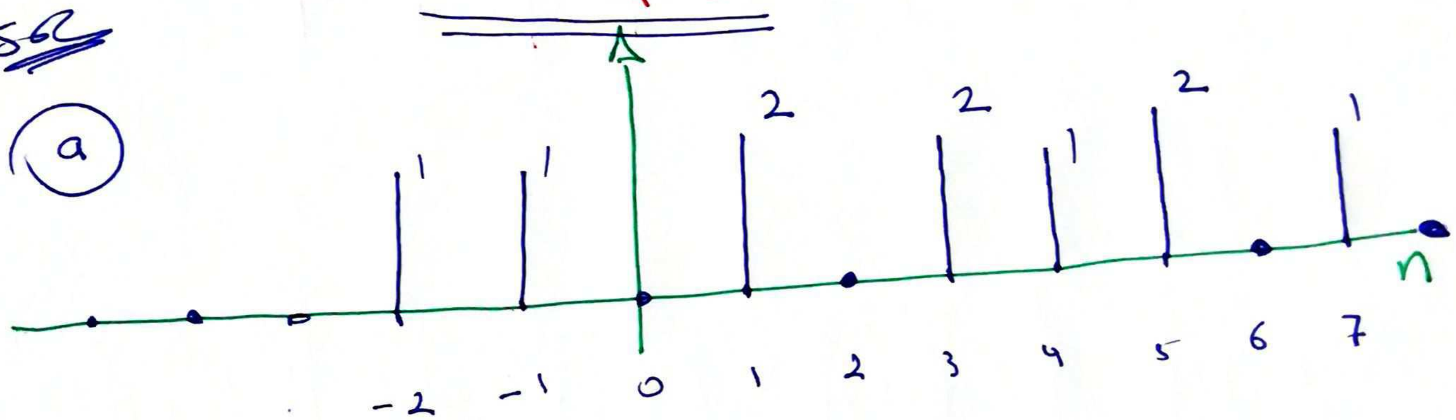
$$X(n) = \begin{cases} 2 & \text{for } n = 1, 3, 5 \\ 1 & \text{for } n = -1, -2, 4, 7 \\ 0 & \text{for otherwise.} \end{cases}$$

(a) Represent $X(n)$ using graphical method.

(b) Decompose (مثل) $X(n)$ into sum (جمع) of weight (الارتفاع) and shifted unit impulse.

Sol

(a)



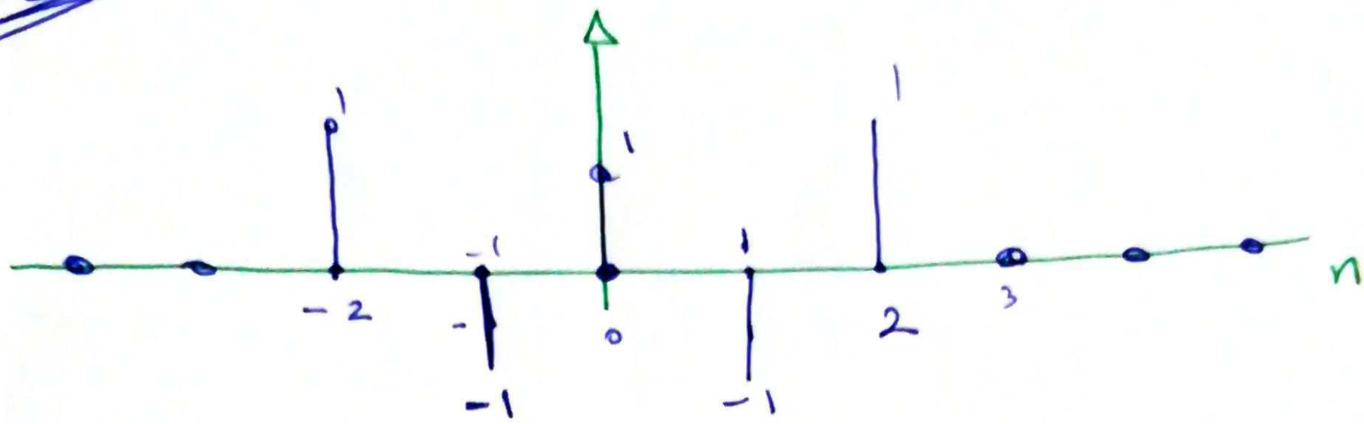
(b)

$$X(n) = 1\delta(n+2) + 1\delta(n+1) + 2\delta(n-1) + 2\delta(n-3) + 1\delta(n-4) + 2\delta(n-5) + 1\delta(n-7).$$

Express the sequence $x(n) = \begin{cases} (-1)^n & -2 \le n \le 2 \\ 0 & \text{elsewhere} \end{cases}$ (4.5)

as sum of scaled and shifted unit sample (unit Impulse).

Sol



$$X(n) = 1\delta(n+2) - 1\delta(n+1) + 1\delta(n)$$

$$-1\delta(n-1) + 1\delta(n-2)$$

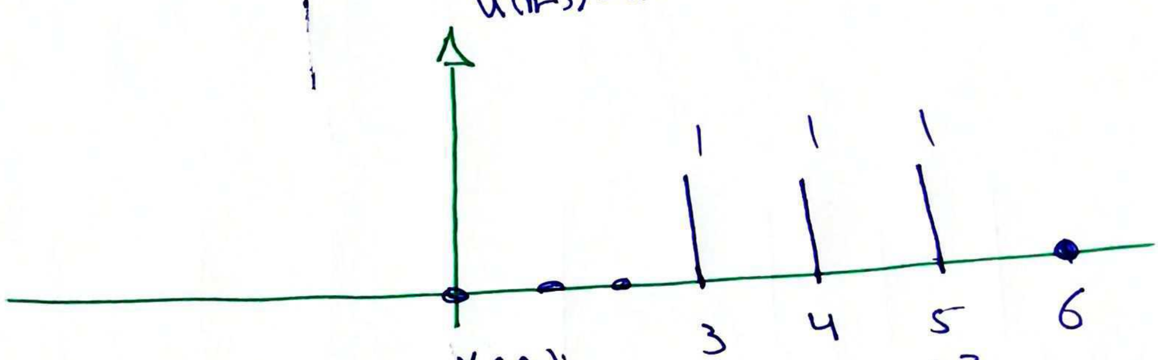
Graphical the discrete input $x(n)$ and the output $y(n)$ Signal

$$x(n) \longrightarrow \boxed{T[\cdot]} \longrightarrow y(n)$$

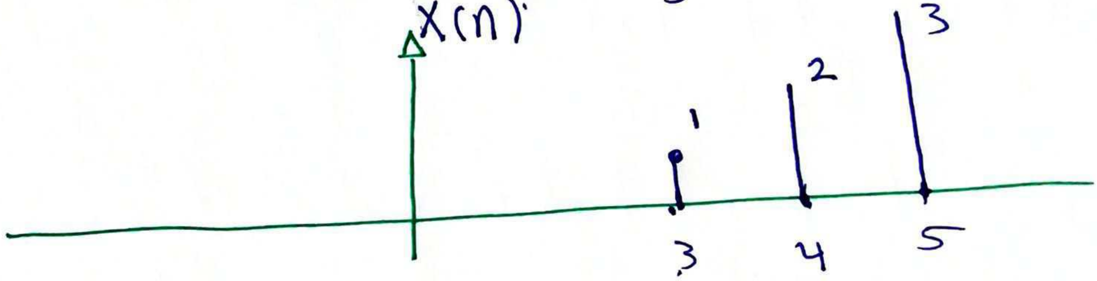
$$x(n) = (n-2)\left[u(n-3) - u(n-6)\right]$$

$$y(n) = x\left(\tfrac{1}{2}n - 2\right)$$

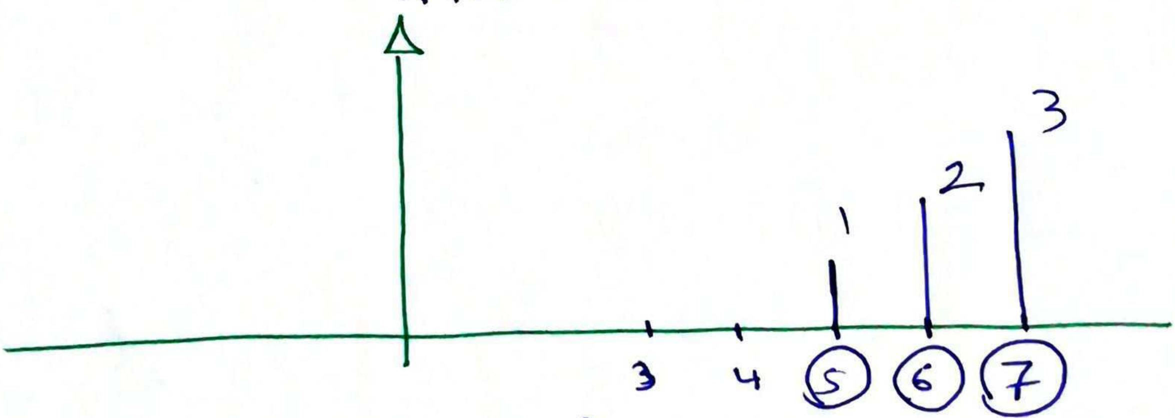
$u(n-3) - u(n-6)$



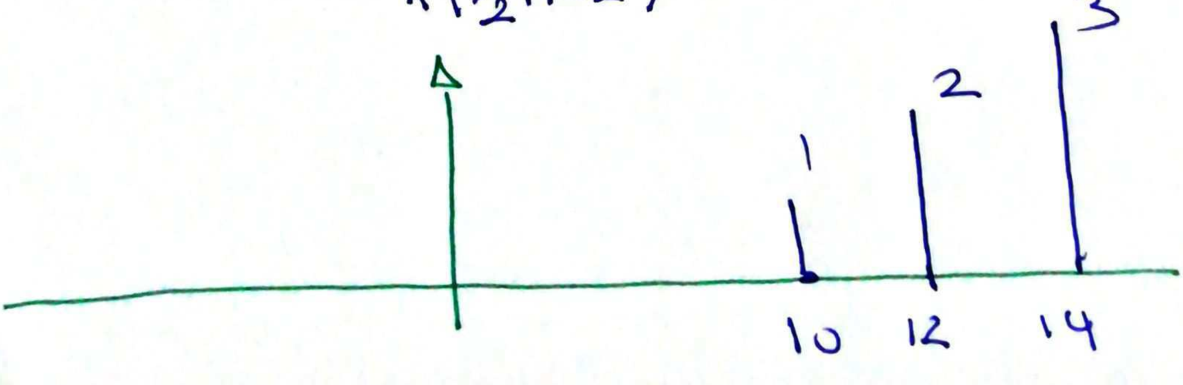
$X(n)$



$x(n-2)$



$x(\frac{1}{2}n-2)$



Priodic and non periodic

① يعطى في السؤال $X(n)$ ← نعوض بدلها $x(n+N)$
ثم نعزل الحد الذي يحتوي على (N) ونساويه بالقانون $2\pi m$
ثم نفرض قيمة m حسب المعطيات

Ex: Determine where the following signals are periodic or not. if periodic find the fundemental period

Ⓐ $x(n) = e^{j7\pi n}$

Sol

$$x(n+N) = e^{j7\pi(n+N)}$$

$$x(n+N) = e^{j(7\pi n + 7\pi N)}$$

نعزل الحد الذي يحتوي على (N)

$$7\cancel{\pi} N = 2\cancel{\pi} M$$ ← قانون

$$N = \frac{2}{7} M$$ نفرض M

$$N = \frac{2}{7} \times 7 = \boxed{2}$$ ← fundemental period

the function is periodic

H.W

determin the fundemental period ?

(a) $x(n) = e^{-j 3 \pi n}$

(b) $x(n) = e^{j 3/2 \pi n}$

(c) $x(n) = e^{-j 3/2 \pi n}$

Ex: find the fundemental period if the function is periodic

$$x(n) = e^{j \frac{3}{5}(n + \frac{1}{2})}$$

$$x(n+N) = e^{j \frac{3}{5}(n + N + \frac{1}{2})}$$

$$x(n+N) = e^{j (\frac{3}{5} n + \frac{3}{5} N + \frac{1}{2})}$$

$$\frac{3}{5} N = 2\pi m \qquad \times 5$$

$$3N = 10 \pi m \Longrightarrow N = \frac{10\pi}{3} M$$

$$N = \frac{10\pi}{\cancel{3}} \times \cancel{3}$$

$N = 10\pi$ the funktion is not periodic

Ex:

(a)

$x(n) = 2\cos\left(\frac{2\pi n}{3}\right)$

Sol

$x(n+N) = 2\cos\left(\frac{2\pi}{3}(n+N)\right)$

$x(n+N) = 2\cos\left(\frac{2\pi n}{3} + \frac{2\pi N}{3}\right)$

$\frac{2\pi}{3} N = 2\pi M$

$\frac{N}{3} = m \Rightarrow N = 3m \Rightarrow$ let $m = 1$

$\therefore \boxed{N = 3} \longrightarrow$ fundimantel period

this function is periodic

Ex: $x(n) = \cos\left(\frac{n}{8} - \pi\right)$

Sol

$x(n+N) = \cos\left(\frac{1}{8}(n+N) - \pi\right)$

$x(n+N) = \cos\left(\frac{n}{8} + \frac{N}{8} - \pi\right)$

$\frac{N}{8} = 2\pi m \Rightarrow N = 16\pi M \Rightarrow \boxed{N = 16\pi}$

the function is not periodic

Ex:

$$x(n) = \cos\left(\frac{5\pi}{9} n + 1\right)$$

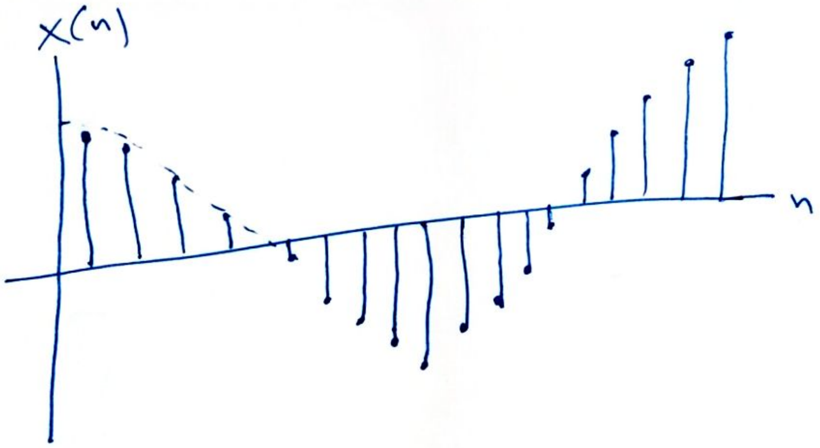
$$x(n+N) = \cos\left(\frac{5\pi}{9}(n+N) + 1\right)$$

$$x(n+N) = \cos\left(\frac{5\pi n}{9} + \frac{5\pi N}{9} + 1\right)$$

$$\frac{5\pi N}{9} = 2\pi m \qquad \times 9$$

$$5\cancel{\pi} N = 18 \cancel{\pi} M \Longrightarrow N = \frac{18\,M}{5} \qquad \text{Let } m = 5$$

$\boxed{N = 18}$ ⟶ the fundemental period



Ex:

Sol $x(n) = \sin\left(\frac{n}{9} - \pi\right)$

$x(n+N) = \sin\left(\frac{n}{9} + \frac{N}{9} - \pi\right)$

$\frac{N}{9} = 2\pi m \Rightarrow \boxed{N = 18\pi M} \rightarrow$ not periodic

Ex: determin the periodic if periodic determine the fundemental period

$x(n) = \sin\left(\frac{\pi}{8} n^2\right)$

Sol

$x(n+N) = \sin\left(\frac{\pi}{8}(n+N)^2\right)$

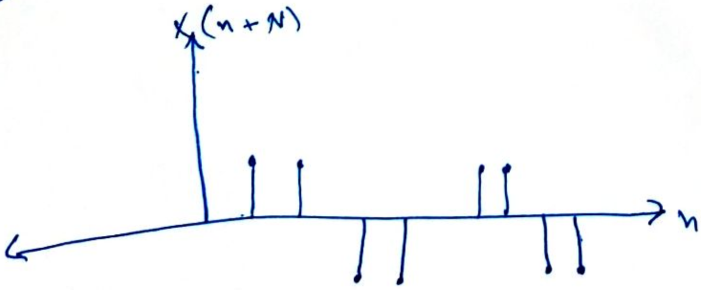
$x(n+N) = \sin\left(\frac{\pi}{8}(n^2 + 2nN + N^2)\right)$

$x(n+N) = \sin\left(\frac{\pi}{8}n^2 + \frac{2\pi nN}{8} + \frac{\pi}{8}N^2\right)$

① $\frac{2\pi N}{8} = 2\pi m \Rightarrow N = 8M \rightarrow$ Let $m = 1$

the fundemental period $\leftarrow \boxed{N = 8}$

② $\frac{\pi}{8}N^2 = 2\pi M \Rightarrow \sqrt{N^2} = \sqrt{16M} \Rightarrow \boxed{N = 4} \rightarrow$ fundemental period



even and odd signals

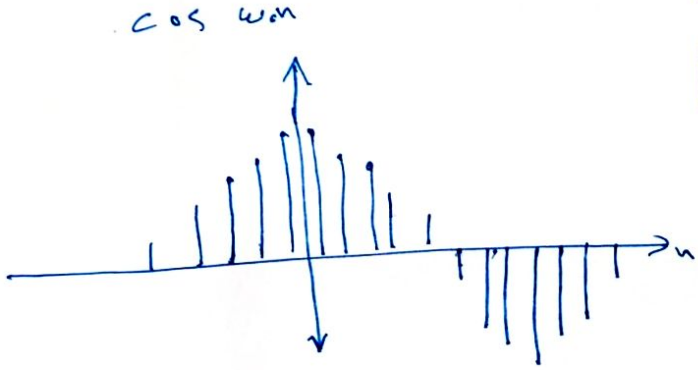
even signal

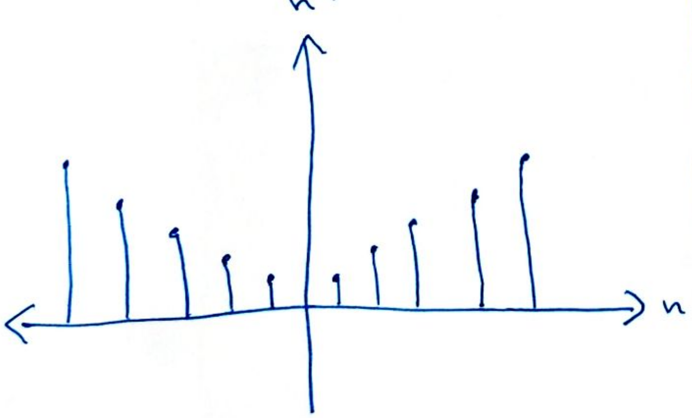
$x(-n) = x(n)$

Ex -

$x(n) = \cos w_0 n$

$x(n) = n^2$





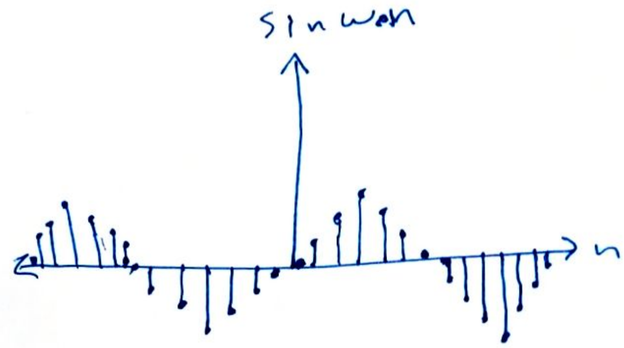
odd signal

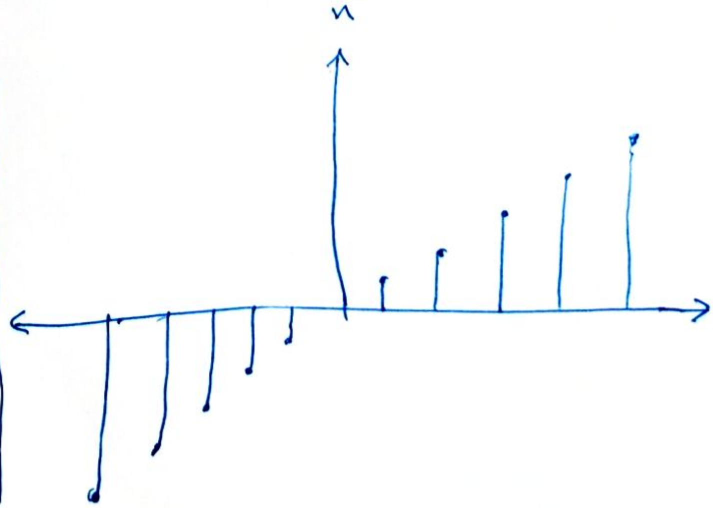
$x(-n) = -x(n)$

Ex

$x(n) = \sin w_0 n$

$x(n) = n^3$

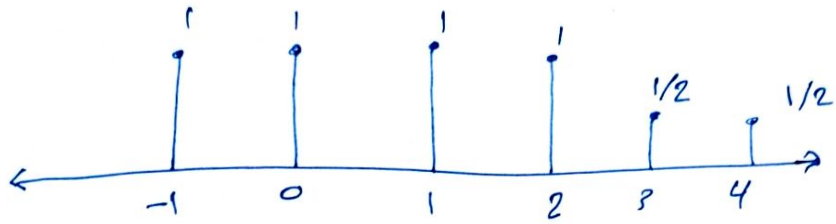




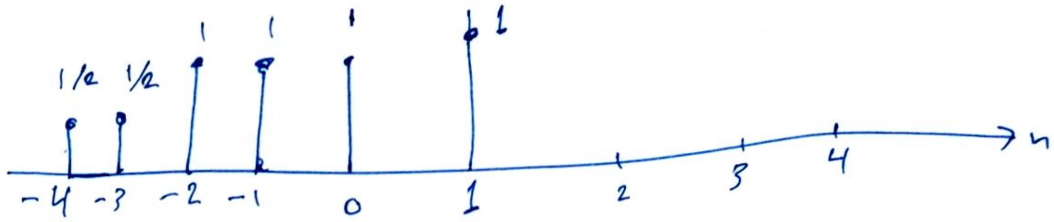
@

$$X_o(n) = \frac{x(n) - x(-n)}{2}$$

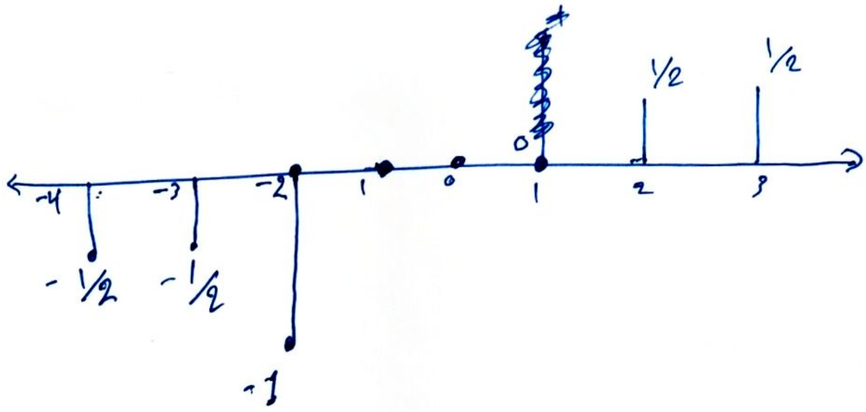
$x(n) =$



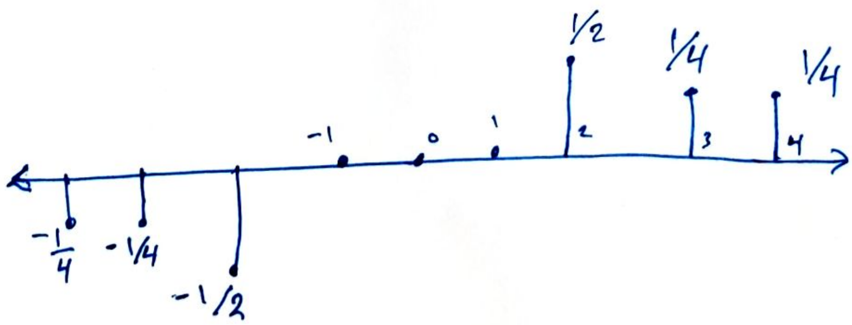
$x(-n) =$



$x(n) - x(-n) =$



$x_o(n) =$

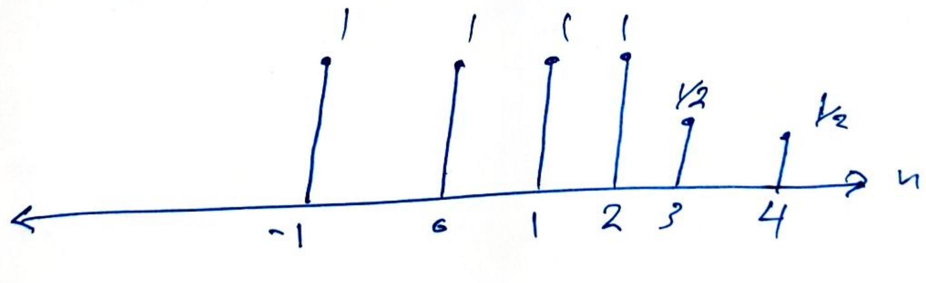


even	odd
$X_e(n) = \frac{X(n)+X(-n)}{2}$	$X_o = \frac{X(n)-X(-n)}{2}$

$\boxed{X(n) = X_e(n) + X_o(n)}$ → equation 1

$\boxed{X(-n) = X_e(n) - X_o(n)}$ → equation 2

Ex: A DT signal is shown in figure sketch and lable carefully each the following signals



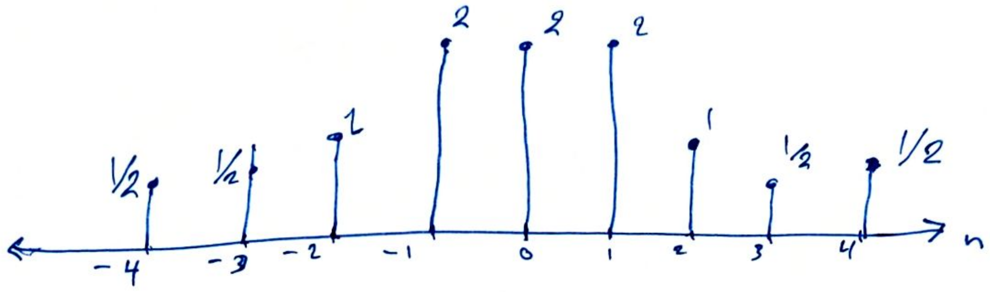
ⓐ odd part of $x(n)$

ⓑ even part of $x(n)$

(b) even part

$$x_e = \frac{1}{2} \quad x(n) + x(-n)$$

$x_e =$



$$x_e(n) = \left\{ \frac{1}{4}, \frac{1}{4}, \frac{1}{2}, 1, 1, 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{4} \right\}$$

$$x_o(n) = \left\{ -\frac{1}{4}, \frac{-1}{4}, -\frac{1}{2}, 0, 0, 0, \frac{1}{2}, \frac{1}{4}, \frac{1}{4} \right\}$$

بعض متطابقات الدوال المثلثية

$$\sin(x+y) = \sin x \cos y + \sin y \cos x$$

$$\sin(x-y) = \sin x \cos y - \sin y \cos x$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$

$$\sin^2 x = \frac{1-\cos(2x)}{2}$$

$$\cancel{\sin} \quad \cos^2 x = \frac{1+\cos(2x)}{2}$$

Energy and power

$$E = \sum_{n=-\infty}^{\infty} (x(n))^2 \qquad P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} (x(n))^2$$

① $$\sum_{n=0}^{\infty} (x(n))^n = \frac{1}{1 - x(n)}$$

② $$\sum_{n=0}^{N} (x(n))^n = \frac{(x(n))^{N+1} - 1}{(x(n)) - 1}$$

Ex: determine whether the following signals energy or power

$$x(n) = \left(\frac{1}{4}\right)^n \cdot u(n)$$

Sol

$$E = \sum_{n=-\infty}^{\infty} (x(n))^2 \Rightarrow E = \sum_{n=-\infty}^{\infty} \left(\left(\frac{1}{4}\right)^n \cdot u(n)\right)^2$$

$$E = \sum_{n=0}^{\infty} \left((0.25)^n\right)^2 \Rightarrow E = \sum_{0}^{\infty} \left((0.25)^2\right)^n$$

$$E = \sum_{0}^{\infty} (0.0625)^n = \frac{1}{1 - 0.0625} = \frac{1}{0.9375}$$

$$\boxed{E = 1.066\ J}$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} (x(n))^2$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left(\left(\frac{1}{4}\right)^n . u(n)\right)^2$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} \left(\left(\frac{1}{4}\right) . u(n)\right) = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{N} (0.0625)^n$$

$$= \lim_{N \to \infty} \frac{1}{2N+1} \frac{(0.0625)^{N+1} - 1}{(0.0625) - 1} \Rightarrow \frac{1}{2\infty + 1} \frac{(0.0625)^{\infty+1} - 1}{0.0625 - 1}$$

$$= \frac{1}{\infty} \frac{(0.0625)^{\infty} - 1}{0.625 - 1} = 0$$

$$P = 0$$

$x(n)$ is energy

Ex: $\sin\left(\frac{\pi}{3} n\right)$

Sol

$$E = \sum_{n=-\infty}^{\infty} (x(n))^2 \Rightarrow E = \sum_{n=-\infty}^{\infty} \left(\sin\left(\frac{\pi}{3} n\right)\right)^2$$

$$E = \sum_{-\infty}^{\infty} \sin^2\left(\frac{\pi}{3} n\right) \Rightarrow E = \frac{1 - \cos(2X)}{2}$$

$$= \sum_{-\infty}^{\infty} \frac{1 - \cos\left(\frac{2\pi}{3} n\right)}{2} \Rightarrow \frac{1}{2} \sum_{-\infty}^{\infty} 1 - \cos(2\pi n/3)$$

$$= \frac{1}{2}\left(1 - \cos(2\pi \infty/3)\right) \Rightarrow E = \frac{1}{2}\left(1 - \cos\infty\right) = \frac{\infty}{2} = \infty$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} (x(n))^2$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{-N}^{N} \left(\sin\left(\frac{\pi}{3} n\right)\right)^2 \Rightarrow P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{-N}^{N} \frac{1 - \cos(2\pi n)}{2}$$

$$P = \frac{1}{2\infty + 1} \sum_{-N}^{N} \frac{1 - \cos(2\pi n)}{2} \Rightarrow P = \frac{1}{2} \quad (1 - \cos(\cancel{2\pi n}))$$

$$P = \frac{1}{2} \ W$$

$x(n)$ is power

Ex :- $x(n) = u(n)$

$$E = \sum_{n=-\infty}^{\infty} (x(n))^2 = \sum_{n=-\infty}^{\infty} (u(n))^2 = \sum_{0}^{\infty} (u(n))^2$$

$$= \sum_{0}^{\infty} (1)^2 = 1 \ J$$

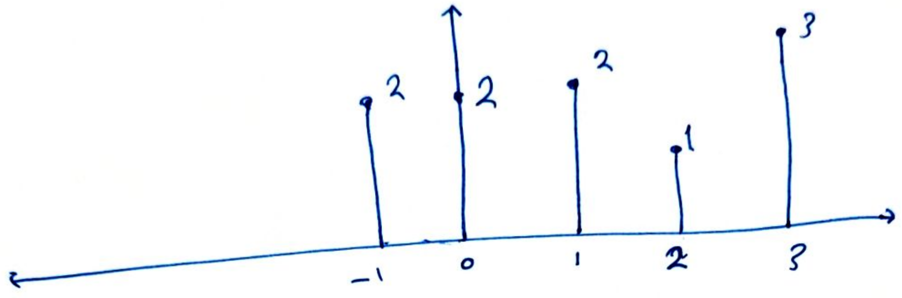
$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} (u(n))^2$$

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=0}^{\infty} (1)$$

$$P = \frac{1}{\infty} (1) = 0 \ W$$

$x(n)$ is energy signal

Ex:- find the energy of signal below



Sol.

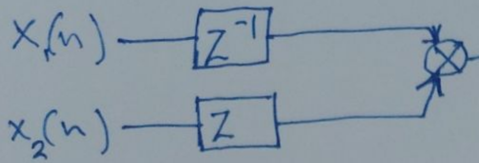
$$E = \sum_{-\infty}^{\infty} (x(n))^2 = \sum_{-1}^{3} (x(n))^2$$

$$= 2^2 + 2^2 + 2^2 + 1^2 + 3^2$$

$$= 4 + 4 + 4 + 1 + 9$$

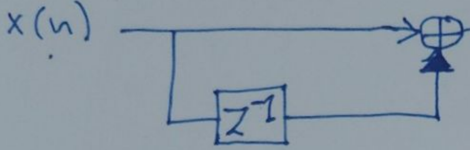
$$= 22 \text{ J}$$

(1)



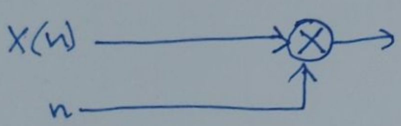
$y(n) = x_1(n-1) * x_2(n+1)$

(2)



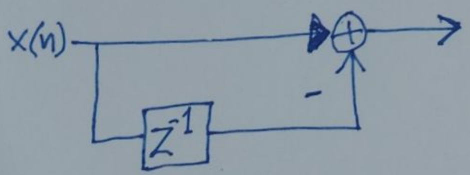
$y(n) = x(n) + x(n-1)$

(3)



$y(n) = n\,x(n)$

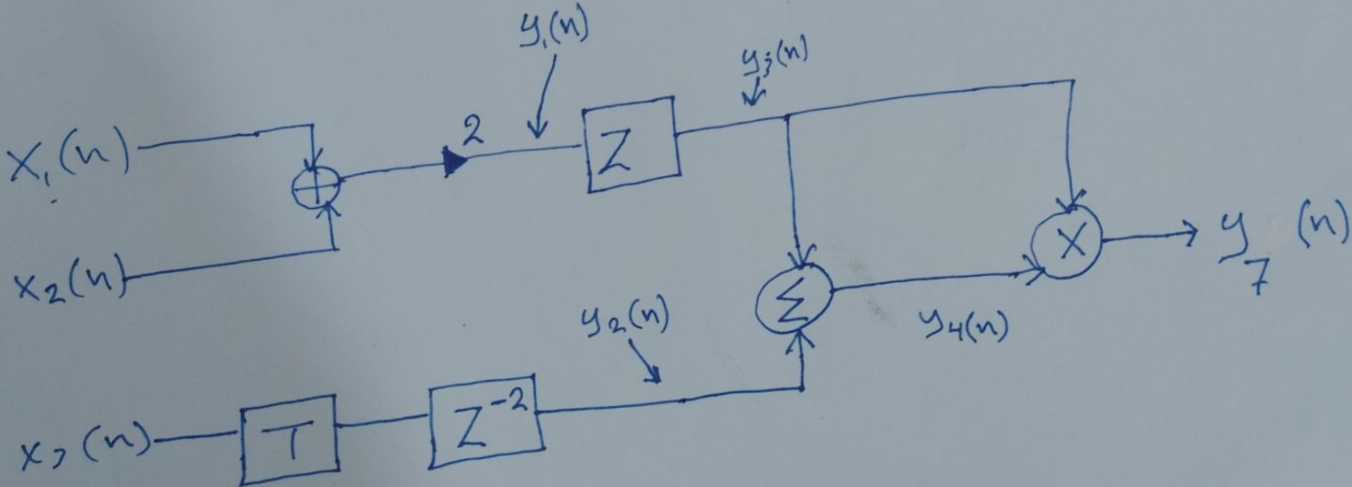
(4)



$y(n) = x(n) - x(n-1)$

(5)

$y(n) = x(-n)$



$$\boxed{y_1(n) = 2\left[x_1(n) + x_2(n)\right]}$$

$$\boxed{y_2(n) = x_3(-n-2)}$$

$$\boxed{y_3(n) = 2x_1(n+1) + 2x_2(n+1)}$$

$$\boxed{y_4(n) = y_2(n) + y_3(n)}$$

$$\boxed{y_7(n) = y_4(n) * y_3(n)}$$

system properties

① Linear or non linear / additive or non additive

② Homogeneous or non Homogeneous

③ Time invariant or time variant

④ causl or non causl

⑤ stable or non stable (unstable)

⑥ memory or memoryLess / dynamic or static

---

linear or non linear

Condition

$$\boxed{T[x_1(n)+x_2(n)] = y_1(n) + y_2(n)}$$

Ex:- $y(n) = x^2(n)$

Sol

$T[x_1(n)+x_2(n)] = y_1(n) + y_2(n)$

$T[x_1(n)+x_2(n)] = T[x_1(n)+x_2(n)]^2$

$T[x_1(n)+x_2(n)] = T[x_1^2(n) + 2x_1(n)x_2(n) + x_2(n)^2]$

$y_1(n) = x_1^2(n)$

$y_2(n) = x_2^2(n)$

$$\boxed{y_1(n) + y_2(n) = x_1^2(n) + x_2^2(n)}$$

$\because\ y_1(n) + y_2(n) \neq T[x_1(n) + x_2(n)]$

$x_1^2(n) + x_2^2(n) \neq [x_1^2(n) + 2x_1(n)x_2(n) + x_2^2(n)]$

$\therefore$ the system is non linear

Ex:-

$y(n) = 2x(n+3) + 3x(n-2)$

Sol

$$\boxed{T[x_1(n) + x_2(n)] = y_1(n) + y_2(n)}$$

$$T[x_1(n) + x_2(n)] = 2[x_1(n+3) + x_2(n+3)] + 3[x_1(n-2) + x_2(n-2)]$$

$y_1(n) = 2x_1(n+3) + 3x_1(n-2)$

$y_2(n) = 2x_2(n+3) + 3x_2(n-2)$

$y_1(n) + y_2(n) = 2x_1(n+3) + 3x_1(n-2) + 2x_2(n+3) + 3x_2(n-2)$

$y_1(n) + y_2(n) = 2[x_1(n+3) + x_2(n+3)] + 3[x_1(n-2) + x_2(n-2)]$

$\because \; T[x_1(n) + x_2(n)] = y_1(n) + y_2(n)$

$\therefore$ the system is linear or additive

Ex:- $y(n) = x(n) \sin\left(\frac{2\pi n}{7} + \frac{\pi}{6}\right)$

<u>Sol</u>

$T[x_1(n) + x_2(n)] = x_1(n) \sin\left(\frac{2\pi n}{7} + \frac{\pi}{6}\right) + x_2(n) \sin\left(\frac{2\pi n}{7} + \frac{\pi}{6}\right)$

$y_1(n) = x_1(n) \sin\left(\frac{2\pi n}{7} + \frac{\pi}{6}\right)$

$y_2(n) = x_2(n) \sin\left(\frac{2\pi n}{7} + \frac{\pi}{6}\right)$

$T[x_1(n) + x_2(n)] = y_1(n) + y_2(n)$

$x_1(n) \sin\left(\frac{2\pi n}{7} + \frac{\pi}{6}\right) + x_2(n) \sin\left(\frac{2\pi n}{7} + \frac{\pi}{6}\right)$

$= x_1(n) \sin\left(\frac{2\pi n}{7} + \frac{\pi}{6}\right) + x_2(n) \sin\left(\frac{2\pi n}{7} + \frac{\pi}{6}\right)$

$\therefore$ the system is linear

Homogeneity or non Homogeneity

$$T[cx(n)] = cT[x(n)]$$

Ex:

$$y(n) = x^2(n)$$

sol

$$T[cx(n)] = c^2x^2(n)$$

$$cT[x(n)] = c\,[x(n)]^2$$

$$\therefore cT[x(n) \neq T[cx(n)]$$

$$cx^2(n) \neq c^2x^2(n)$$

$\therefore$ the system is not homogeneous

Ex:-

$$y(n) = 3x(n) + 2x(n-6)$$

Sol/

$$T[cx(n)] = 3cx(n) + 2cx(n-6)$$

$$cT[x(n)] = c[3x(n) + 2x(n-6)]$$

$$\because cT[x(n)] = T[cx(n)]$$

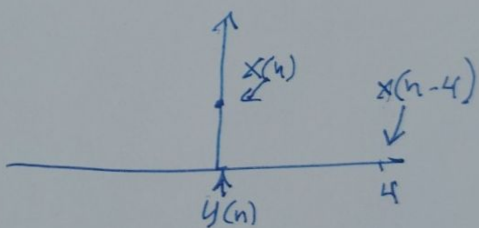
$\therefore$ the system is homogeneous

---

causal or non causal

Ex:-

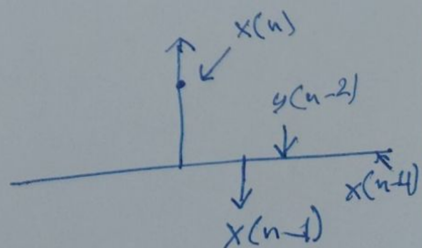
$$y(n) = x(n) + x(n-4)$$



the system is causal

Ex:-

$$y(n-2) = x(n-2) - x(n-1) + x(n)$$



the system is not causal

dynamic or static

memory or memory less

Ex÷ $y(n) = x^2(n) \longrightarrow$ memory less $\longrightarrow$ static

$y(n) = x(n+2) \longrightarrow$ memory $\longrightarrow$ dynamic

$y(n-1) = x(n-1) \longrightarrow$ memoryless $\longrightarrow$ static

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Varian or invariant

step ①

$x(n) \longrightarrow \boxed{\text{delay by } n_0} \longrightarrow \boxed{T[x(n)]} \longrightarrow y(n)$

step ②

$x(n) \longrightarrow \boxed{T[x(n)]} \longrightarrow \boxed{\text{delay by } n_0} \longrightarrow y'(n)$

if $y(n) = y'(n) \longrightarrow$ time invariant

Ex: for given discrete tim system represented by

$y(n) = x(n) + x(-n)$ determin the system time invariant or not

Sol

step ①

$$x(n)+x(-n) \longrightarrow \boxed{\text{delay by } n_0} \xrightarrow{x(n-n_0)+x(-n-n_0)} \boxed{T[x(n)]} \longrightarrow \boxed{x(n-n_0)+x(-n-n_0)}$$

step ②

$$x(n)+x(-n) \longrightarrow \boxed{T[x(n)]} \xrightarrow{x(n)+x(-(n))} \boxed{\text{delay by } n_0} \longrightarrow x(n-n_0)+x(-(n-n_0))$$

$$\boxed{= x(n-n_0)+x(-n+n_0)}$$

$\because$ step ① $\neq$ step ②

$$x(n-n_0)+x(-n-n_0) \neq x(n-n_0)+x(-n+n_0)$$

$\therefore$ the system is Time variant

Stable versus unstable system

BIBo [Bonded Input Bounded output]

Ex: $y(n) = y^2(n-1) + 4\delta(n)$

$y(-1) = 0$

Sol/

$y(0) = y^2(0-1) + 4\delta(0) = y^2(-1) + 4 = (0)^2 + 4 = 4$

$y(1) = y^2(1-1) + 4\delta(1) = y^2(0) + 4*0 = 4^2 = 16$

$y(2) = y^2(2-1) + 0 = y^2(1) = (16)^2 = 256$

$y(3) = y^2(3-1) = y^2(2) = (256)^2 = 65536$

the system is unstable (unbound o/p)

Ex: $y(n) = u(n) + 1$

$y(0) = 1 + 1 = 2$

$y(1) = 1 + 1 = 2$

$y(2) = 1 + 1 = 2$

$y(3) = 1 + 1 = 2$

the system is stable (bound o/p)