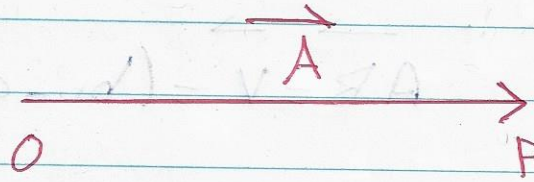
م.م حسن حسني

Engineering mathematics

Vector analysis

Vector :- is a quantity having both magnitude and direction such as displacement, velocity, force and acceleration.



Scalar :- is a quantity having magnitude but no direction, energy, mass, length, time, temperature and any real number.

finding the vector from point A to point B

$$A = (a_1, a_2, a_3)$$

$$B = (b_1, b_2, b_3)$$

$$\overrightarrow{AB} = \overrightarrow{V} = (b_1 - a_1, b_2 - a_2, b_3 - a_3)$$

مريم حسن حسني

finding the vector from point B to poin A

$$\overrightarrow{BA} = \overrightarrow{V} = (a_1 - b_1, a_2 - b_2, a_3 - b_3)$$

example:. find the vector from point

$A = (2, -7, 0)$ to point $B = (1, -3, -5)$

Solution :.

$$\overrightarrow{AB} = \overrightarrow{V} = (b_1 - a_1, b_2 - a_2, b_3 - a_3)$$

$$= (1-2, -3+7, -5-0)$$

$$= (-1, 4, -5)$$

example:. find the vector from point

$C = (2, -7, 0)$ to point $D = (2, -7, 0)$

Solution:.

$$\overrightarrow{CD} = \overrightarrow{V} = (d_1 - c_1, d_2 - c_2, d_3 - c_3)$$

$$= (2-1, -7+3, 0+5)$$

$$= (1, -4, 5)$$

مريم حسن حسني

H.W

1) Find the vector from point $Z=(0,6,1)$ to point $W=(3,-2,-4)$.

2) Find the vector from point $N=(-1,2,9)$ to point $M=(0,-4,0)$.

---

length of the vector

The length of the vector $\vec{U}=(a_1,a_2,a_3)$

$$\|\vec{V}\|=|\vec{V}|=\sqrt{a_1^2+a_2^2+a_3^2}$$

example: Find the length of the vector

1) $\vec{A}=(3,-5,10)$

2) $\vec{U}=\left(\frac{1}{\sqrt{5}},\frac{-2}{\sqrt{5}}\right)$

3) $\vec{W}=(0,0)$

4) $\vec{V}=(1,0,0)$

م.م حسن حسني

Solution:-

1) $|\vec{A}| = \sqrt{a_1^2 + a_2^2 + a_3^2} = \sqrt{(3)^2 + (-5)^2 + (10)^2}$

$= \sqrt{9 + 25 + 100} = \sqrt{134}$

2) $|\vec{U}| = \sqrt{U_1^2 + U_2^2 + U_3^2} = \sqrt{\left(\frac{1}{\sqrt{5}}\right)^2 + \left(\frac{-2}{\sqrt{5}}\right)^2}$

$= \sqrt{\frac{1}{5} + \frac{4}{5}} = \sqrt{\frac{5}{5}} = \sqrt{1} = 1$

3) $|\vec{W}| = \sqrt{W_1^2 + W_2^2} = \sqrt{(0)^2 + (0)^2} = 0$

Zero Vector

4) $|\vec{V}| = \sqrt{V_1^2 + V_2^2 + V_3^2} = \sqrt{(1)^2 + (0)^2 + (0)^2}$.

$= \sqrt{1} = 1$ Unit Vector

H.W

find the length of the Vector

1) $\vec{Q} = (7, -5, 0)$

2) $\vec{R} = (1, -2, 4)$

أ.م.م حسن حسين

Unit vector

Unit vector:- is a vector having unit magnitude, if $\vec{A}$ is a vector with magnitude $A \neq 0$, then $\frac{\vec{A}}{|\vec{A}|}$ is unit vector having the direction as $\vec{A}$.

$$\vec{u} = \frac{\vec{A}}{|\vec{A}|}$$

$$|\vec{A}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

example:- find the unit vector in the direction of $\vec{A} = \vec{i} - 2\vec{j} + 3\vec{k}$

solusion:-

$$|\vec{A}| = \sqrt{a_1^2 + a_2^2 + a_3^2} = \sqrt{(1)^2 + (-2)^2 + (3)^2}$$

$$= \sqrt{1+4+9} = \sqrt{14}$$

$$\vec{u} = \frac{\vec{A}}{|\vec{A}|} = \frac{\vec{i} - 2\vec{j} + 3\vec{k}}{\sqrt{14}}$$

example:- find the vector $\vec{A}$ directed from

point $(3, -5, 2)$ to point ~~(0, -2, 0)~~ $(1, -3, 1)$

and find the unit vector along $\vec{A}$.

solution:-

let $P_1 = (3, -5, 2)$, $P_2 = (1, -3, 1)$

$$\vec{A} = P_2 - P_1$$

$$\vec{A} = (1-3)\vec{i} + (-3+5)\vec{j} + (1-2)\vec{k}$$

$$\vec{A} = -2\vec{i} + 2\vec{j} - \vec{k}$$

$$|\vec{A}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

$$= \sqrt{(-2)^2 + (2)^2 + (-1)^2}$$

$$= \sqrt{4 + 4 + 1} = \sqrt{9} = 3$$

$$\overrightarrow{u} = \frac{\overrightarrow{A}}{|\vec{A}|} = \frac{-2\vec{i} + 2\vec{j} - \vec{k}}{3}$$

$$= \frac{-2}{3}\vec{i} + \frac{2}{3}\vec{j} - \frac{1}{3}\vec{k}$$

م.م حسن حنشي فليح

example: find unit vector in the direction of the vector from $P=(5,7,-1)$ to $Q=(2,9,-2)$.

Solution:

$$\overrightarrow{PQ}=(2-5)\vec{i}+(9-7)\vec{j}+(-2+1)\vec{k}$$

$$=-3\vec{i}+2\vec{j}-\vec{k}$$

$$|\overrightarrow{PQ}|=\sqrt{(-3)^2+(2)^2+(-1)^2}$$

$$=\sqrt{9+4+1}=\sqrt{14}$$

$$\therefore |\overrightarrow{PQ}|=\sqrt{14}$$

$$|\vec{u}|=\frac{\overrightarrow{PQ}}{|\overrightarrow{PQ}|}=\frac{-3\vec{i}+2\vec{j}-\vec{k}}{\sqrt{14}}$$

م.م حسن حسن

Vector Algebra operations

let $u = (u_1, u_2, u_3)$

$v = (v_1, v_2, v_3)$

1) Addition

$$u + v = (u_1 + v_1, u_2 + v_2, u_3 + v_3)$$

2) Subtraction

$$u - v = (u_1 - v_1, u_2 - v_2, u_3 - v_3)$$

3) Scalar multiplication

$$k \cdot u = (k u_1, k u_2, k u_3)$$

Note:- 1) if $k > 0 \Rightarrow k u$

عندما نضرب المتجه u بقيمة موجبة المتجه يبقى بنفس الاتجاه،

2) if $k < 0 \Rightarrow k u$

عندما نضرب المتجه u بقيمة سالبة المتجه يكون بعكس الاتجاه،

Proov that

$$|ku| = |k|\,|u|$$

Solution:-

$$|ku| = \sqrt{(ku_1)^2 + (ku_2)^2 + (ku_3)^2}$$

$$= \sqrt{k^2 (u_1^2 + u_2^2 + u_3^2)} = \sqrt{k^2}\ \sqrt{u_1^2 + u_2^2 + u_3^2}$$

$$= |k| \cdot |u|$$

ح.ح حسن حسني

$\therefore\ |kU| = |k|\,|U|$ يحقق

Properties of Vector operations

let U, V, W and a, b, c

1) $U + V = V + U$
2) $(U + V) + W = U + (V + W)$
3) $U + 0 = U$
4) $U + (-U) = 0$
5) $0 \cdot U = 0$
6) $1 \cdot U = U$
7) $a(bU) = (ab)U$
8) $a(U + V) = aU + aV$
9) $(a + b)U = aU + bU$

حسن حسني ٢٠٢٠

example:- let

$u = (-1, 3, 1)$

$v = (4, 7, 0)$

find

1) $u+v$
2) $u-v$
3) $|u+v|$
4) $|u-v|$
5) $2u+3v$
6) $|\frac{1}{3} \cdot u|$

solution:-

1) $u+v = (-1, 3, 1) + (4, 7, 0)$
$= (-1+4, 3+7, 1+0)$
$= (3, 10, 1)$

2) $u-v = (-1, 3, 1) - (4, 7, 0)$
$= (-1-4, 3-7, 1-0)$
$= (-5, -4, 1)$

3) $|u+v| = \sqrt{(3)^2 + (10)^2 + (1)^2} = \sqrt{9+100+1}$
$= \sqrt{110}$

4) $|u-v| = \sqrt{(-5)^2 + (-4)^2 + (1)^2} = \sqrt{25+16+1}$
$= \sqrt{42}$

حسن حنش ٢٠٩

5) $2U + 3V = 2(-1, 3, 1) + 3(4, 7, 0)$

$= (-2, 6, 2) + (12, 21, 0)$

$= (10, 27, 2)$

6) $|\frac{1}{3} \cdot U| = |\frac{1}{3}| \cdot |U| = \frac{1}{3}\sqrt{(-1)^2 + (3)^2 + (1)^2}$

$= \frac{1}{3}\sqrt{1 + 9 + 1} = \frac{1}{3}\sqrt{11}$

H.W

example: let

$U = (4, 2, 1)$

$V = (0, -3, 5)$

Find

1) $|V - U|$

2) $4U - 2V$

3) $|\frac{5}{6} V|$

4) $3V - 2U$

م.م حسن حسّن

example:- let $U = (-1, 2, 0)$

$V = (2, 5, 6)$

$W = (-1, -2, 5)$

proov that

1) $U + V = V + U$

2) $(U+V) + W = U + (V+W)$

solution:-

1) $U + V = V + U$

$U + V = (-1, 2, 0) + (2, 5, 6)$

$= (1, 7, 6)$

$V + U = (2, 5, 6) + (-1, 2, 0)$

$= (1, 7, 6)$

$\therefore U + V = V + U$

2) $(U+V) + W = U + (V+W)$

$U + V = (-1, 2, 0) + (2, 5, 6)$

$= (1, 7, 6)$

$(U+V) + W = (1, 7, 6) + (-1, -2, 5)$

$= (0, 5, 11)$

$V + W = (2, 5, 6) + (-1, -2, 5)$

$= (1, 3, 11)$

$U + (V+W) = (-1, 2, 0) + (1, 3, 11)$

$= (0, 5, 11)$

$\therefore (U+V) + W = U + (V+W)$

م.م حسن حشاش
متجه الحلقات

Vector fields

$\vec{F}(x,y) = P(x,y)\vec{i} + Q(x,y)\vec{j}$

$P, Q =$ Scalar field

$\vec{F}(x,y,z) = P(x,y,z)\vec{i} + Q(x,y,z)\vec{j} + R(x,y,z)\vec{k}$

The law of vector fields

1) $\vec{F}(x,y) = -y\vec{i} + x\vec{j}$

2) $\vec{F}(x,y,z) = 2x\vec{i} - 2y\vec{j} - 2x\vec{k}$

example: find the vector field

1) $\vec{F}\left(\frac{1}{2}, -\frac{1}{2}\right)$

2) ~~$\vec{F}(x,y,z)$~~ $\vec{F}(1,-3,2)$

Solution:

1) $\vec{F}(x,y) = -y\vec{i} + x\vec{j}$

$= -\left(-\frac{1}{2}\right)\vec{i} + \frac{1}{2}\vec{j}$

$= \frac{1}{2}\vec{i} + \frac{1}{2}\vec{j}$

مريم حسن حنش

2) $\vec{F}(x,y,z) = 2x\vec{i} - 2y\vec{j} - 2x\vec{K}$

$$= 2(1)\vec{i} - 2(-3)\vec{j} - 2(1)\vec{K}$$

$$= 2\vec{i} + 6\vec{j} - 2\vec{K}$$

example: find the vector field and Draw

$$\vec{F}(x,y) = [(1,0),(0,1),(-1,0),(0,-1),(1,1),(-1,1),(-1,-1),(1,-1)]$$

Solution:

$$\vec{F}(x,y) = -y\vec{i} + x\vec{j}$$

(x,y)	$-y\vec{i} + x\vec{j}$
$(1,0)$	$\langle 0,1\rangle$
$(0,1)$	$\langle -1,0\rangle$
$(-1,0)$	$\langle 0,-1\rangle$
$(0,-1)$	$\langle 1,0\rangle$
$(1,1)$	$\langle -1,1\rangle$
$(-1,1)$	$\langle -1,-1\rangle$
$(-1,-1)$	$\langle 1,-1\rangle$
$(1,-1)$	$\langle 1,1\rangle$

$\vec{F}(1,0) = -(0)\vec{i} + (1)\vec{j} = (0)\vec{i} + \vec{j}$

$\vec{F}(0,1) = -(1)\vec{i} + (0)\vec{j} = -\vec{i} + (0)\vec{j}$

$\vec{F}(-1,0) = -(0)\vec{i} + (-1)\vec{j} = (0)\vec{i} - \vec{j}$

$\vec{F}(0,-1) = -(-1)\vec{i} + (0)\vec{j} = \vec{i} + (0)\vec{j}$

$\vec{F}(1,1) = -(1)\vec{i} + (1)\vec{j} = -\vec{i} + \vec{j}$

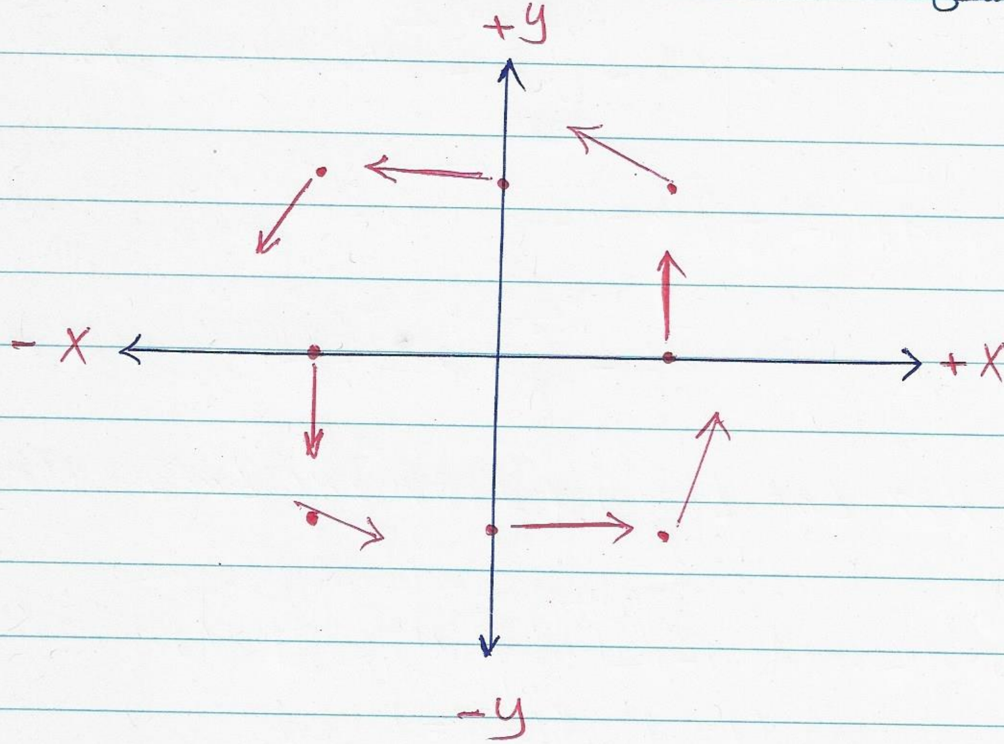
$\vec{F}(-1,1) = -(1)\vec{i} + (-1)\vec{j} = -\vec{i} - \vec{j}$

$\vec{F}(-1,-1) = (-1)\vec{i} + (-1)\vec{j} = \vec{i} - \vec{j}$

$\vec{F}(1,-1) = -(-1)\vec{i} + (1)\vec{j}$

$= \vec{i} + \vec{j}$

م.م حسن حنش



Dot Product

Net: There are two ways to express vectors both of which are correct.

1) $\vec{A} = (x, y, z)$

2) $\vec{A} = a\vec{i} + b\vec{j} + c\vec{K}$

Not: the Dot product of vectors gives a number not a new vector, for example 1, 2, 3, ...

$\vec{A} \cdot \vec{B} = C$

م.م حسن حنش

$$\cos\alpha = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}|\cdot|\vec{B}|} \Rightarrow \alpha = \cos^{-1}\frac{\vec{A}\cdot\vec{B}}{|\vec{A}|\cdot|\vec{B}|}$$

example:. find the Dot product

$$\vec{A} = 4\vec{i} + \vec{j} + 4\vec{k}$$

&

$$\vec{B} = 3\vec{i} + 2\vec{j} + 4\vec{k}$$

solution:.

$$\vec{A}\cdot\vec{B} = (4\times3) + (1\times2) + (4\times4)$$
$$= 12 + 2 + 16$$
$$= 30$$

example:. if $\vec{A} = 2\vec{i} + 3\vec{j} + \vec{k}$

$\vec{B} = 4\vec{i} - 2\vec{j} - 2\vec{k}$

show that $\vec{A}$ and $\vec{B}$ are orthogonal

solution:.

$$\vec{A}\cdot\vec{B} = (2\times4) + (3\times-2) + (1\times-2)$$
$$= 8 - 6 - 2 = 0$$

$$\therefore \vec{A} \perp \vec{B}$$

م.م حسين حسني

example:: find the angle between

$$\vec{A} = 2\vec{i} + 2\vec{j} - \vec{k} \quad \text{and} \quad \vec{B} = 6\vec{i} - 3\vec{j} + 2\vec{k}$$

solution::

$$\cos\alpha = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| \cdot |\vec{B}|} \Rightarrow \alpha = \cos^{-1} \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| \cdot |\vec{B}|}$$

$$\vec{A} \cdot \vec{B} = (2 \times 6) + (2 \times (-3)) + ((-1) \times 2)$$

$$= 4$$

$$|\vec{A}| = \sqrt{(2)^2 + (2)^2 + (-1)^2} = 3$$

$$|\vec{B}| = \sqrt{(6)^2 + (-3)^2 + (2)^2} = 7$$

$$\therefore \alpha = \cos^{-1} \frac{4}{21}$$

$$\Rightarrow \alpha = \cos^{-1} 0.1905$$

$$\Rightarrow \alpha = 79^\circ$$

م.م حسن حنش

Properties of Dot Product

1) $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

2) $\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A}\vec{B} + \vec{A}\vec{C}$

3) $m \cdot (\vec{A} \cdot \vec{B}) = (m \cdot \vec{A}) \cdot \vec{B} = (m \cdot \vec{B}) \cdot \vec{A} = (\vec{A} \cdot \vec{B}) \cdot m$

where m is scalar.

4) $\vec{i} \cdot \vec{i} = \vec{j} \cdot \vec{j} = \vec{K} \cdot \vec{K} = 1$

$\vec{i} \cdot \vec{j} = \vec{j} \cdot \vec{K} = \vec{K} \cdot \vec{i} = 0$

5) if vectors $\vec{A}$ and $\vec{B}$ are orthogonal then

$\vec{A} \cdot \vec{B} = 0$

Vector projections

مساقط المتجه

law

$$\text{Proj}_V U = \frac{U \cdot V}{|V|^2} \cdot V$$

Not: $|U| \cos\theta$ is called the scalar component of U in the direction V

ملاحظة: تسمى المركبات العددية للمتجه U بالنسبة للاتجاه V ،

$$|U| \cos\theta = \frac{U \cdot V}{|V|} = U \cdot \frac{V}{|V|}$$

م.م حسين حنشي

ex: Find the vector projection of $u = 6\vec{i} + 3\vec{j} + 2\vec{K}$ onto $V = \vec{i} - 2\vec{j} - 2\vec{K}$ and the scalar component of u onto V

solution:

$$\text{proj}_V u = \frac{u \cdot V}{|V|^2} \cdot V$$

$$\vec{u} \cdot \vec{V} = (6 \times 1) + (3 \times (-2)) + (2 \times (-2))$$

$$= 6 - 6 - 4 = -4$$

$$|V|^2 = \left(\cancel{\sqrt{(1)^2 + (-2)^2 + (-2)^2}}\right)^{\cancel{2}}$$

$$= 1 + 4 + 4 = 9$$

$$\text{Proj}_V u = \frac{-4}{9}(\vec{i} - 2\vec{j} - 2\vec{K})$$

$$= \frac{-4}{9}\vec{i} + \frac{8}{9}\vec{j} + \frac{8}{9}\vec{K}$$

$$|u| \cos\theta = \frac{u \cdot V}{|V|} = \frac{-4}{3}$$

□

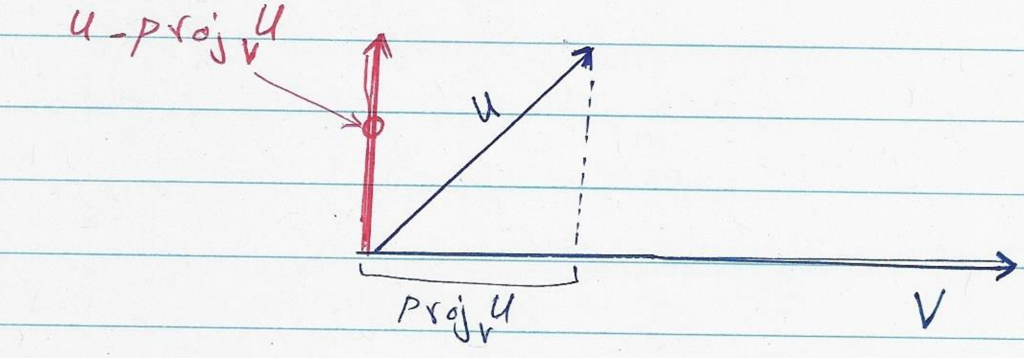
Note:

$$\vec{V} \cdot \vec{V} = |\vec{V}|^2$$

م.م حسين حنشي

example: verify that vector $u - proj_v u$ is orthogonal to projection vector $proj_v u$

solution:



$$(u - proj_v u) \cdot v = \left(u - \frac{u \cdot v}{|v|^2} v\right) \cdot v$$

$$= u \cdot v - \left(\frac{u \cdot v}{|v|^2} v\right) v$$

$$v \cdot v = |v|^2$$

$$= u \cdot v - \frac{u \cdot v}{|v|^2} v \cdot v$$

$$= u \cdot v - \frac{u \cdot v}{\cancel{|v|^2}} \cdot \cancel{|v|^2}$$

$$= u \cdot v - u \cdot v$$

$$= 0$$

$$\therefore (u - proj_v u) \perp proj_v u$$